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Imperfect Competition and Leverage in the Banking Sector

Abstract

This paper evaluates the role of financial frictions and banking intermediation in the real business cycle in Brazil, specifically in the spread charged by Brazilian banks. We estimate a dynamic general equilibrium model (DSGE) for Brazil that incorporates a Cournot banking sector where banks accumulate capital subject to a capital adequacy requirement. Our findings show that the spread is more significant in a scenario with imperfect banking competition and bank capital accumulation. Amplified countercyclical spread, which arises from a joint effect between the elasticity of loans varying over time, the market power of banks, and their cost of capitalization, tends to amplify the response of output, investment, consumption, and physical capital in the presence of adverse shocks. We also show that most of the spread increase in Brazil is due to financial shocks, mainly after 2008. The financial shocks that increase the spread contribute for the most part to the fall in accumulated output in Brazil.

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1 Introduction

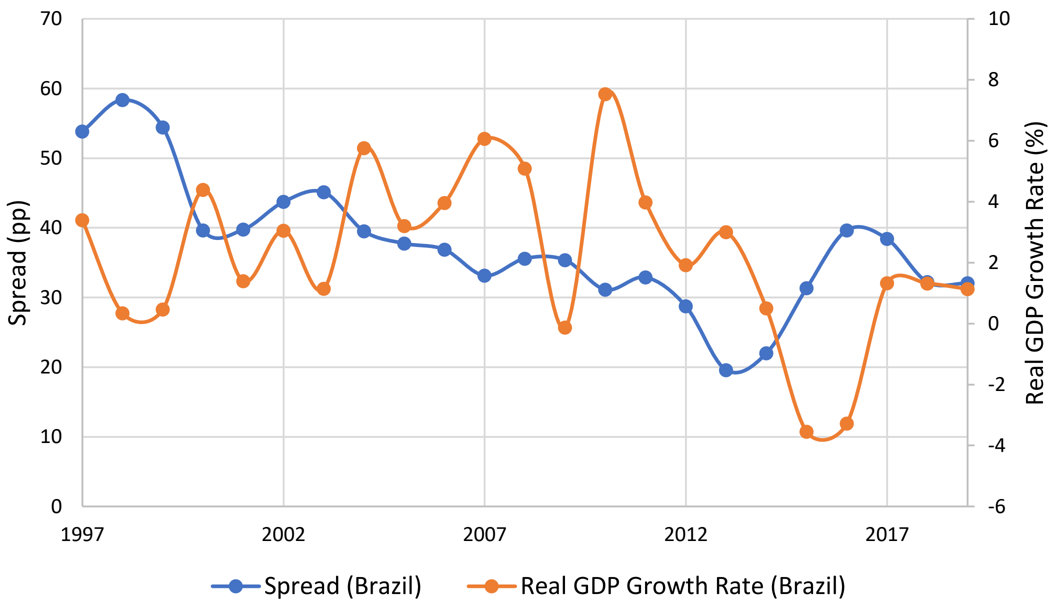
There is an increasing focus on incorporating financial frictions into dynamic stochastic general equilibrium (DSGE) models following the 2008 financial crisis. Most of the existing literature studies financial frictions in the context of amplifying aggregate fluctuations and often models financial frictions using an agency problem and a perfectly competitive banking sector. However, in reality, the banking sector tends to be imperfectly competitive. In recent decades, according to Joaquim et al. (2019), the world banking system is characterized by high concentration and, on average, the five largest banks in each country dominate a significant fraction of assets. In Brazil, the situation is no different. There was a substantial increase from 50% to over 85% in the share of large banks assets in 1990-2016.

The high banking concentration in the Brazilian market, as suggested by Joaquim et al. (2019), contributes to the high value of the bank's spread (the difference between the rates charged on loans and the rates paid on deposits) and a decrease in the number of loans. The effects are not restricted to the credit market but expand to the real economy, causing a drop in employment and output. Few firms will expand their investments and hire employees because of higher interest rates on loans taken out at banks, which hinders the development of business activities.

This paper evaluates the role of financial frictions and banking intermediation in the real business cycle in Brazil, specifically in the spread charged by Brazilian banks. We estimate a dynamic general equilibrium model (DSGE) for Brazil that incorporates a Cournot banking sector where banks accumulate capital subject to a capital adequacy requirement. Our findings show that the spread is more significant in a scenario with imperfect banking competition and bank capital accumulation. Amplified countercyclical spread, which arises from a joint effect between the elasticity of loans varying over time, the market power of banks, and their cost of capitalization, tends to amplify the response of output, investment, consumption, and physical capital in the presence of adverse shocks. We approach productivity, collateral, financial, and investment shocks. Figure 1 shows the countercyclical spread using country-level data for Brazil from 1997 to 2019.

How the spread changes, in our model, in response to shocks depend on the elasticity of demand for loans to the loan rate and the banks' capitalization cost. Based on the model, the demand for loans becomes more inelastic when the expected future prices of capital decrease and the expected marginal product of capital increases. More clearly, in the presence of binding collateral constraint, low expected asset prices after negative productivity and investment shocks indicate a reduced borrowing capacity of entrepreneurs. A high expected marginal product means borrowers operate below the optimal scale due to the more tightly binding borrowing constraint. As a result, entrepreneurs are more financially constrained, leading to more inelastic demand for loans. Besides, a negative shock that reduces the collateral of the entrepreneurs causes a reduction in the elasticity of the demand for loans, making borrowers more financially constrained. The lower elasticity of loan demand provides banks with market power an incentive to charge a higher loan rate, which generates a higher spread.

Figure 1: Spread and real GDP growth in Brazil from 1997 to 2019



Note: The annual spread (in percentage points) from the World Bank is the difference between the lending rate (charged by banks on loans to the private sector) and deposit rate (offered by commercial banks on three-month deposits). The graph plots the unweighted average spread for Brazil (blue line) over time. The orange line corresponds to the annual real GDP growth rate for Brazil.

About the financial shock, banks accumulate capital from retained earnings while keeping their capital/assets ratio as close as possible to a target level (exogenously given). This target level can be derived from mandatory capital requirements for banking activity (such as those explicitly established in the Basel Accords) or the country's Central Bank. Through this leverage ratio and the identity bank's balance sheet, the bank's capital influences both the loans issued and the setting of loan rates. In this way, financial shocks that hit bank capital are amplified by the cost of capitalization of banks and introduce essential feedback loops between the real and financial sides of the economy.

Our model contributes to the literature by joining the imperfect banking competition and bank stress channels to explain the countercyclical character of the spread in a recession and see which channel is most important for Brazil. The imperfect banking competition channel stands out for the market power of banks that can regulate the spread. On the other hand, the bank stress channel refers to the bank lending channel, and the spread depends on bank capital, so financial shocks that affect banks' balance sheets change the spread behavior.

The paper has three main objectives. First, studying how the accumulation of capital made by large banks can affect the countercyclical behavior of spread in an environment composed of financially restricted entrepreneurs. Second, verify the effects of adverse shocks (productivity, collateral, investment, and financial) on the real economy. Third, see how perfect banking competition can reduce banks' market power and reduce the effects of adverse shocks in an interbank market where banks can accumulate capital.

2 Theoretical Reference

The starting point for the incorporation of credit market frictions in the dynamic stochastic general equilibrium models (DSGE) as well as the study of the relationships between financial markets and the real economy dates back to the last decades especially after the 2008 financial crisis. The definition of financial frictions (or credit market frictions) can be understood as the difficulty of agents to carry out transactions due to information asymmetry, agency costs or collateral constraints. The presence of these market failures acts as a financial accelerator and amplifies output, inflation and interest rates fluctuations, more specifically, the credit market has a significant impact on the real economy.

The part of the literature that models credit market frictions through the principal-agent problem between borrowers and lenders to generate a financial accelerator can be seen in Kiyotaki and Moore (1997), Christiano et al. (2010), Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). In these models, borrowers' financial conditions tend to worsen during a crisis and agency costs problems become more severe, resulting in difficulties in obtaining new external financing and amplifying the effects of shocks that could negatively affect borrowers' financial conditions.

In this line of study, Kiyotaki and Moore (1997) show how credit constraints affect economic activity throughout the business cycle. Lenders cannot obliges borrowers to repay their debts unless those debts are secured by assets that are pledged, i.e., durable goods in this economy serve as collateral for loans. However, the credit limits that borrowers may require are restricted by the prices of collateral assets which in turn are affected by the size of the credit limit. Thus, the interaction between asset prices and credit limits becomes a powerful transmission mechanism through which the effects of shocks are amplified and transmitted to other sectors of the economy. Transmission occurs because firms may experience a temporary negative shock to productivity that reduces their net worth and make them unable to borrow more. The most financially constrained firms are forced to reduce their investments and this also affects them in the next period as they earn less revenue their net worth falls and, due to credit constraints, they again reduce investment and thus affect aggregate output.

Christiano et al. (2010) create a more complex environment than the environment seen in Kiyotaki and Moore (1997). In this economy, banks provide loans to finance firms' capital requirements from deposits that pay their holders a nominal interest rate determined by contract. However, loans pose a risk to banks because firms investment returns are subject to idiosyncratic shocks that can lead to their insolvency and inability to repay the loan. For this reason, banks protect themselves against credit risk and information asymmetry by charging a premium above the risk-free rate they pay to individuals depositing their wealth with banks. The authors conclude that financial shocks are responsible for a substantial portion of economic fluctuations and the risk shock is dominant. The wealth shock alters the value of total equity in the economy and the purchasing power of investors. The risk shock already affects investment returns and determines investors' propensity to invest and banks' propensity to lend.

Already Gertler and Kiyotaki (2010) develop a model for thinking about credit market frictions and the effects on economic activity in the context of the 2008 financial crisis. In addition, they highlight two aspects of the crisis that have not been fully captured in previous studies. First, the 2008 financial crisis represented a significant drop in financial intermediation (private banks expanding credit across sectors) and much of the previous macroeconomic literature on financial frictions emphasized only credit market constraints on borrowers. Second, to combat the crisis both monetary and fiscal authorities in many countries, including the US, have employed a number of unconventional policies that involve some form of direct lending in the credit markets.

Different than Kiyotaki and Moore (1997) and Christiano et al. (2010), in Gertler and Kiyotaki (2010) financial intermediaries (banks) are able to evaluate and monitor borrowers, which makes lenders' credit flow to the non-financial sectors of the economy more efficient. They present an agency problem that potentially restricts banks' ability to raise funds from depositors and introduces a difference between loan and deposit rates (spread). During a crisis, the spread charged by financial intermediaries with loss-making funds increases substantially and, in turn, increases the cost of credit that non-financial borrowers face which may also affect real activity.

In the same line of Gertler and Kiyotaki (2010), in Gertler and Karadi (2011) financial intermediaries face constraints that are endogenously determined. In this environment, there is a Central Bank using an unconventional monetary policy to counteract the negative effects of a financial crisis. Unconventional monetary policy is defined as the Central Bank's credit expansion to compensate for a disruption of private financial activity. During the crisis, the financial constraint on private intermediaries increases, making room for the benefits of Central Bank intervention. In all these papers where financial intermediation is explicitly modeled, the banking sector is perfectly competitive. However, in another segment of the macroeconomic literature the banking sector is monopolistically competitive and made up of small banks. Stand out Gerali et al. (2010), Andrés and Arce (2012) and Hafstead and Smith (2012).

The results found by Gerali et al. (2010) are that tight interest rates mitigate the effects of monetary policy while financial intermediation increases the propagation of credit supply shocks. In addition, shocks from the banking sector explain most of the contraction in economic activity in the 2008 financial crisis, while macroeconomic shocks played a minor role, and unexpected destruction of banking capital could have substantial effects on economic activity. Bank lending margins depend on the elasticities of demand for loans relative to interest rates, the degree of interest rate rigidity, and the ratio of capital to bank assets. Bank balance sheet constraints link the business cycle that affects bank profits to capital accumulation. Variation in banks' capital accumulation affects the supply and cost of lending to other sectors of the economy.

Already Andrés and Arce (2012) study how the degree of bank competition affects the transmission of shocks in the economy and thus its overall stability. Banks determine the optimal rates charged on loans according to the effects of their pricing policies and the volume of funds required by each borrower. Banking competition reduces the margin between lending and deposit rates which gives rise to two competing effects. First, lower lending margins imply greater leverage which tends to broaden the short-term response of housing prices, consumption and output. However, lower lending margins also promote a faster recovery of borrowers' net worth and thus their ability to raise funds and produce in the face of an adverse shock. The nature of the shock that hits the economy and the time horizon considered determines which force is dominant.

Lastly, Hafstead and Smith (2012) expand the standard financial accelerator model of Bernanke et al. (1999) with the inclusion of a heterogeneous and monopolistically competitive banking sector. With these characteristics of the banking sector, it is possible to measure the impact of credit supply and demand shocks on the real economy. The authors show that the inclusion of the uncompetitive banking sector mitigates the impact of credit frictions on macroeconomic fluctuations affecting both the magnitude and persistence of non-financial shocks. Moreover, the impact of intermediation costs on the investment imply that it is the most important shock in explaining the variation of the real variables. Therefore, a monetary policy that reacts to financial market spreads may improve economic performance relative to a monetary policy that reacts only to macroeconomic variables.

Despite the results of this segment of the literature, Li (2019) says that the evidences shows that the banking sector tends to be dominated by only a few large banks (OECD and EU countries). Li (2019) contributes to the macroeconomic literature in two segments. First, she incorporates imperfect competition through a Cournot banking sector with an agency problem that generates collateral constraints, i.e., banks behave like an oligopoly and ask for collateral to lend. Second, she shows that in the presence of firms with bind collateral constraint, the uncompetitive banking sector tends to amplify output, investment and physical capital responses after a contractionary monetary shock and also after a negative collateral shock in which firm asset prices are reduced.

This result differs from the attenuation effect found in the existing literature (Gerali et al. (2010), Andrés and Arce (2012) and Hafstead and Smith (2012))and one exception is Cuciniello and Signoretti (2014) because the authors find that monopolistic banking competition can amplify aggregate fluctuations after monetary policy contractionary but this result is supported strategic interaction between banks with market power and the Central Bank's inflation target. After the adverse shock (monetary or collateral), according to Li (2019), firms' ability to borrow decreases and they become more financially constrained which makes borrowing demand more inelastic, i.e., firms accept a higher interest rate charged by banks. Market-power banks can take advantage of firms' financial constraints by reducing the amount of loans available on the market. So, in equilibrium a higher interest rate is charged on the loans, increasing the banks' profits. Therefore, imperfect bank competition tends to be an important mechanism for the propagation of macroeconomic shocks, especially when firms are financially constrained and the degree of bank competition is low.

Finally, a third part of the literature seen in Bernanke et al. (1999), Pesaran et al. (2006) and most recently in Pesaran and Xu (2016) models the relationship between bank loans and firm investment decisions, specifically how the risks of offering credit impact the prices of loans. However, their approach differs from models that consider collateral constraints such as Kiyotaki and Moore (1997), Christiano et al. (2010), Gertler and Kiyotaki (2010). In the latter, collateral restrictions are introduced in order to oblige borrowers to pay their debts and in most cases, in equilibrium, make it impossible for a firm to default.

The Pesaran and Xu (2016)'s model differs from the Bernanke et al. (1999)'s model because idiosyncratic shocks affect firm productivity rather than return on capital which establishes a direct relationship between credit risk and productivity. Households bear some of the risk of firms' default due to the adverse technology shock because they own firms' stocks. Otherwise, all the negative effects of a firms' default would be on banks, generating an excessive bank spread. Results found by Pesaran and Xu (2016) are that the probability of default increases with firms' leverage growth and increasing economic uncertainty. In addition, a positive credit shock (increase in the level of loans relative to the number of deposits) made by the banking sector or the Central Bank causes an expansion in available capital and, consequently, in investment and in the output of the economy. These results are consistent with Xu (2012).

3 Model

Our reference model follows Li (2019) and Gerali et al. (2010). However, it has several modifications. There is Cournot banking competition in Li (2019), but no bank capital accumulation exists. Banks return all profits to households. Gerali et al. (2010) is monopolistic banking competition, and banks accumulate all profits (zero dividend policy). In our model, there is Cournot banking competition, and banks return a portion of earnings in the form of dividends to households and accumulate the remainder in bank capital.

3.1 Households

There is a continuum of identical infinitely-lived households of unit mass that maximize the following expected utility function:

$$\max_{\{c_t,l_t,d_t\}} E_t \sum_{s=0}^{\infty} \beta^s \left[\ln(c_{t+s}) + \phi_l \ln(1 - l_{t+s})\right] \tag{1}$$

which depends on consumption c_{t+s} and labor supply l_{t+s}, with $\beta \in (0, 1)$ being the subjective discount factor of households. In each period, households consumes c_t, saves d_t (in terms of real final consumption) and offer l_t hours of labor. Time is normalized to 1 and $(1 - l_t)$ can be defined as the amount of leisure of households in period-t and ϕ_l is the relative utility weight on leisure time.

Assume that in this economy, the households own the capital production sector, the retail firms, and are bank shareholders. Also, assume that there are no risk-free bonds, so in equilibrium, households keep only bank deposits d_t. Nominal deposits d_{t-1} saved in period $t - 1$ yield a gross nominal interest rate R^d_{t-1} at the beginning of period t. In addition to deposit gains $R^d_{t-1}d_{t-1}$, households have income from work $w_t l_t$, profits from the capital formation sector Γ^{CP}_t, retail firms Γ^R_t and dividends div^B_t paid by banks. Thus, the representative household has the following budget constraint:

$$c_t + d_t = \frac{R^d_{t-1} d_{t-1}}{\pi_t} + w_t l_t + \Gamma^{CP}_t + \Gamma^R_t + div^B_t \tag{2}$$

which $\pi_t \equiv \frac{p_t}{p_{t-1}}$ denote the gross infation rate and p_t is the unit price of final consumption good. We will denote the Lagrange multiplier associated with the representative household budget constraint by λ_t and the first order conditions with respect to consumption c_t (3), labor supply l_t (4), and bank deposits d_t (5) can be written as:[1]

$$\lambda_t = \frac{1}{c_t} \tag{3}$$

$$\lambda_t w_t = \frac{\phi_l}{(1 - l_t)} \tag{4}$$

$$\lambda_t = \beta E_t \left[\lambda_{t+1} \frac{R^d_t}{\pi_{t+1}}\right] \tag{5}$$

where the equation (5) is the intertemporal Euler equation, which can also be written as:

$$1 = E_t \left[\Lambda_{t,t+1} \frac{R^d_t}{\pi_{t+1}}\right] \tag{6}$$

where $\Lambda_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t} = \beta \frac{u'(c_{t+1})}{u'(c_t)} = \beta \frac{c_t}{c_{t+1}}$ is the stochastic discount factor in period t for real payoffs in period $t + 1$, with $u(c_t) = \ln(c_t)$.

[1] The households' optimization problem is described in Appendix A.

3.2 Entrepreneurs

There is a continuum i of perfectly competitive entrepreneurs of unit mass that have some capital endowment in the initial period. In period $t-1$, entrepreneurs acquire physical capital k_{t-1} from capital producers at the real price q_{t-1} and in the period t hire labor l_t from households that will be used as inputs to produce a wholesale good $y_t^w(i)$ through constant-returns-to-scale Cobb-Douglas production technology:

$$y_t^w = z_t(k_{t-1}(i))^\alpha (l_t(i))^{1-\alpha} \tag{7}$$

where $\alpha \in (0,1)$ is output elasticity of physical capital. The wholesale good $y_t^w(i)$ produced in period t is then sold to retailers at a nominal price $p_t^w(i)$, who produce the final consumption good y_t sold at a nominal price p_t. The total factor productivity z_t follows an autoregressive process AR(1):

$$\ln(z_t) = \psi_z \ln(z_{t-1}) + \varepsilon_t^z \tag{8}$$

where $\psi_z \in (0,1)$ reflects the persistence of z_t and ε_t^z is a productivity shock with variance σ_z^2.

Let β^E denote the subjective discount factor of entrepreneurs. It is assumed that $\beta^E < \beta$ to ensure that entrepreneurs are net borrowers and households are net savers in steady-state and its neighborhood, following Iacoviello (2005). The objective of entrepreneurs is to maximize their expected lifetime utility:

$$E_t \sum_{s=0}^{\infty} (\beta^E)^s \ln(c_{t+s}^E) \tag{9}$$

subject to a budget constraint:

$$c_t^E + q_t k_t + w_t l_t + \frac{R_{t-1}^b b_{t-1}}{\pi_t} = \frac{y_t^w}{x_t} + q_t(1-\delta)k_{t-1} + b_t \tag{10}$$

where $x_t \equiv \frac{p_t}{p_t^w}$ denotes the mark-up of the price of the final consumption good y_t over the price of the wholesale good y_t^w. The loans taken out in the banking sector in the period t are represented by b_t and R_t^b denotes the interest rate that entrepreneurs will be paid for these loans. At the end of the period t, entrepreneurs can sell non-depreciated capital $(1-\delta)k_{t-1}$ to capital producers at price q_t, where δ is the depreciation rate of physical capital. The wholesale good y_t^w produced in period t is then sold to the retailers at the price p_t^w. On the expenditure side of entrepreneurs, the outflow of funds is given by consumption c_t^E, cost of capital investiment $q_t k_t$, wage payments $w_t l_t$ and gross loan interest payments $\frac{R_{t-1}^b b_{t-1}}{\pi_t}$.

An agency problem is introduced, following to Kiyotaki and Moore (1997), assuming costly debt enforcement. If entrepreneurs fail to honor their debts, banks may confiscate part of entrepreneurs' assets. Assuming that physical capital k_t can be used as collateral assets, let $m_t^k \in (0,1)$ denote the fraction of physical capital collateral that banks can confiscate if entrepreneurs fail to repay their loans. Consequently, the maximum amount that entrepreneurs can borrow is such that the gross nominal debt interest payment $R_t^b b_t$ is equal to the expected value of their assets that banks can recover $m_t^k E_t[q_{t+1}(1-\delta)k_t]$ after entrepreneurs do not make their payments. Thus, the entrepreneurs are also subject to a borrowing constraint:

$$b_t \leq m_t^k E_t\left[\frac{q_{t+1}(1-\delta)k_t\pi_{t+1}}{R_t^b}\right] \tag{11}$$

The pledgeability ratio m_t^k is subject to the collateral shocks and follows an autoregressive process AR(1):

$$\ln(m_t^k) = \psi_{m^k}\ln(m_{t-1}^k) + \varepsilon_{m_t^k} \tag{12}$$

where $\psi_{m^k} \in (0,1)$ indicates the persistence of the m_t^k and $\varepsilon_{m_t^k}$ is the collateral shock with variance $\sigma^2_{m^k}$. Since the pledgeability ratio is defined as a loan-to-value ratio (amount of loan divided by the value of the collateral), the collateral shock can also be defined as a macropudential policy shock.

Let $\lambda_{1,t}^E$ and $\lambda_{2,t}^E$ denote the lagrangian multipliers associated with the budget constraint (10) and the borrowing constraint (11), respectively. Then, the first-order conditions of entrepreneurs' optimization problem in relation to entrepreneurs' consumption c_t^E (13), labor demand l_t (14), loan demand b_t (15), and capital demand k_t (16) are:[2]

$$\lambda_{1,t}^E = \frac{1}{c_t^E} \tag{13}$$

$$w_t = (1-\alpha)\frac{y_t^w}{x_t l_t} \tag{14}$$

$$\lambda_{2,t}^E = \lambda_{1,t}^E - \beta^E E_t\left[\lambda_{1,t+1}^E\frac{R_t^b}{\pi_{t+1}}\right] \tag{15}$$

$$\lambda_{1,t}^E q_t = \beta^E E_t\left[\lambda_{1,t+1}^E\left(\alpha\frac{y_{t+1}^w}{x_{t+1}k_t} + (1-\delta)q_{t+1}\right)\right] + \lambda_{2,t}^E E_t\left[\frac{m_t^k(1-\delta)q_{t+1}\pi_{t+1}}{R_t^b}\right] \tag{16}$$

Combining the equations (13) and (15), we get the following expression in the steady-state:

$$\lambda_2^E = \frac{1}{c^E}\left(1-\beta^E\frac{R^b}{\pi}\right) \tag{17}$$

The value of steady-state of the gross real interest rate $\frac{R^d}{\pi}$ is determined by the households' subjective discount factor such that $\frac{R^d}{\pi} = \frac{1}{\beta}$, according to Euler equation (6). To ensure that the borrowing constraint is always binding in the steady-state, λ_2^E must be positive, which implies $\beta^E < \beta$. The heterogeneity in the β and β^E guarantees that

[2]The entrepreneurs' optimization problem is described in Appendix B.

entrepreneurs are net borrowers in the steady-state.[3]

Based on the budget constraint (10), entrepreneur's net worth n_t in period t, after the productivity shock has been realized realized and the output y_t^w produced, is defined by:

$$n_t = \frac{y_t^w}{x_t} - w_t l_t + q_t(1-\delta)k_{t-1} - \frac{R_{t-1}^b b_{t-1}}{\pi_t} \tag{18}$$

where $q_t(1-\delta)k_{t-1}$ is the total value of the capital stock and $\frac{R_{t-1}^b b_{t-1}}{\pi_t}$ is the loan interest payment at the beginning of period t. Then, the budget constraint (10) can be written in terms of n_t:

$$c_t^E + q_t k_t = n_t + b_t \tag{19}$$

which implies that the entrepreneur finances consumption c_t^E and the purchase of new capital k_t through bank loans b_t and retained earnings n_t. Under the assumption of log utility, c_t^E is a fixed proportion of the accumulated profits n_t:

$$c_t^E = (1-\beta^E)n_t \tag{20}$$

The real loan demand b_t also can be written as the total purchasing cost of new capital in excess of the internal financing or savings $\beta^E n_t$:[4]

$$b_t = q_t k_t - \beta^E n_t \tag{21}$$

where $\beta^E n_t$ is the portion of retained earnings that is not consumed and can be used to purchase new capital.

Note that the binding borrowing constraint (11) determines the market loan demand and it implies the inverse relation between the equilibrium loan rate R_t^b and loan quantity b_t. In the perfect banking competition scenario, loan rate R_t^b is given by the gross deposit rate R_t^d, thus b_t is determined. With imperfect banking competition, each individual bank determine the amount of b_t and consequently affect the R_t^b. In particular, for a given asset prices q_{t+1} and π_{t+1}, a higher loan rate R_t^b corresponds to a lower loan quantity b_t and affects the demand for capital.

3.3 Capital Producers

There is a continuum of perfectly competitive capital producers of unit mass that are introduced to obtain an explicit expression of the capital price q_t (Gambacorta and Signoretti (2014)). Capital producers buy non-depreciated capital $(1-\delta)k_{t-1}$ from firms and also buy final consumption good i_t from retailers to produce new capital k_t at the

[3]In the literature, its standard approach to assume $\beta^E < \beta$ to ensure that the borrowing constraint permanently binds in the steady-state and its neighborhood, as long as the size of shocks are sufficiently small (Iacoviello (2005), Andrés and Arce (2012), Gerali et al. (2010)).

[4]In the presence of binding budget constraint (19).

end of period t:

$$k_t = (1-\delta)k_{t-1} + i_t \tag{22}$$

where χ represents the adjustment cost of investment, i_t is also gross investiment and k_t is the new produced capital that will be sold back to the firms at the real price q_t. The capital k_t will be used in the production of the wholesale good in period $t+1$. Following Christiano et al. (2005), assume that old capital can be converted into new capital at a one-to-one rate subject to a quadratic investment adjustment cost $f\left(\frac{i_t}{i_{t-1}}\right) = \frac{\chi}{2}\left(\frac{i_t s_t^{qk}}{i_{t-1}} - 1\right)^2$ with $f(1) = f'(1) = 0$, $f''(1) > 0$. The adjustment cost specification shows that few units of new capital can be produced from one investment unit whenever $\frac{i_t}{i_{t-1}}$ deviates from its unitary value of the steady-state. In addition, $\chi > 0$ reflects the magnitude of the adjustment cost and s_t^{qk} is a total factor productivity of the investment i_t that follows an autorregressive AR(1):

$$\ln(s_t^{qk}) = \psi_{s^{qk}} \ln(s_{t-1}^{qk}) + \varepsilon_t^{qk} \tag{23}$$

where $\psi_{s^{qk}}$ measures the degree of persistence of s_t^{qk} and ε_t^{qk} is a investment productivity shock with variance $\sigma_{s^{qk}}^2$.

The capital producer chooses the level of gross investment i_t that maximizes the sum of the expected discounted future profits made from the sale of the new capital k_t at the price q_t minus the payment of input costs $(q_t(1-\delta)k_{t-1} + i_t)$ and investment adjustament cost $f\left(\frac{i_t}{i_{t-1}}\right) i_t$:

$$\max_{\{i_t, k_t\}} E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left[q_t k_t - q_t(1-\delta)k_{t-1} - i_t - \frac{\chi}{2}\left(\frac{i_t s_t^{qk}}{i_{t-1}}\right)^2 i_t \right] \tag{24}$$

where $\Lambda_{t,t+s} = \beta^s \frac{u'(c_{t+s})}{u'(c_t)}$ is the stochastic discount factor, since households own the capital producers. Replacing (22) in (24), the objective function can be simplified to:

$$\max_{\{i_t\}} E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left\{ (q_t - 1)i_t - q_t \frac{\chi}{2}\left(\frac{i_t s_t^{qk}}{i_{t-1}} - 1\right)^2 i_t \right\} \tag{25}$$

The capital producer's problem returns the relation (26) to the capital price q_t taking the first order condition with respect to i_t:[5]

$$q_t = 1 + \frac{\chi}{2}\left(\frac{i_t s_t^{qk}}{i_{t-1}} - 1\right)^2 + \chi\left(\frac{i_t s_t^{qk}}{i_{t-1}} - 1\right)\left(\frac{i_t}{i_{t-1}}\right) s_t^{qk} - \chi E_t \left\{ \Lambda_{t,t+1} \left(\frac{i_{t+1} s_{t+1}^{qk}}{i_t} - 1\right)\left(\frac{i_{t+1}}{i_t}\right)^2 s_{t+1}^{qk} \right\} \tag{26}$$

In the steady-state, the real capital price q_t is equal to one since $i_{t-1} = i_t = i_{t+1}$. All profits Γ_t^{CP} made outside the steady-state $(q \neq 1)$ by capital producers sector return to households where $\Gamma_t^{CP} = (q_t - 1)i_t - \frac{\chi}{2}\left(\frac{i_t s_t^{qk}}{i_{t-1}} - 1\right)^2 i_t$.

[5]The capital producers' optimization problem is described in Appendix C.

3.4 Retailers

A continuum of retailers of unit mass, indexed by i, buy the wholesale good $y_t^w(i)$ at a nominal price $p_t^w(i)$ from firms and use it as the only input to produce differentiated retail goods costlessly. Nominal rigidity is introduced by assuming the retailers are monopolistically competitive and set prices à la Rotemberg (1982).[6] Each retailer i produces a different variety $y_t(i)$ and charges a nominal price $p_t(i)$ for the differentiated product. The output of the final consumption good y_t is a constant elasticity of substitution (CES) composite of all the different varieties produced by the retailers (using the Dixit and Stiglitz (1977) framework):

$$y_t = \left[\int_0^1 y_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}} \tag{27}$$

where $\epsilon > 1$ is is the elasticity of intratemporal substitution between different varieties.

Each retailer i then sells his unique variety, applying a markup over the wholesale price, taking into account the demand that he faces characterized by a stochastic price-elasticity ϵ_t^y. Retailers' prices are sticky and are indexed to a combination of past and steady-state inflation, with relative weights parameterized by ι_p. Whenever retailers want to change their price beyond what indexation allows, they face a quadratic adjustment cost parameterized by κ_π. Then, retailers must choose $\{p_t(i)\}_{t=0}^{\infty}$ to maximize profits given by:[7]

$$\Gamma^R = E_t \sum_{t=0}^{\infty} \Lambda_{t,t+s} \left[p_t(i)y_t(i) - p_t^w(i)y_t(i) - \frac{\kappa_\pi}{2}\left(\frac{p_t(i)}{p_{t-1}(i)} - \pi_{t-1}^{\iota_p}\overline{\pi}^{1-\iota_p}\right)^2 p_t y_t\right] \tag{28}$$

subject to a downward sloping demand coming from consumers maximization of a consumption aggregator:

$$y_t(i) = y_t \left(\frac{p_t(i)}{p_t}\right)^{-\epsilon_t^y} \tag{29}$$

The price-elasticity ϵ_t^y follows an autorregressive $AR(1)$:

$$\ln(\epsilon_t^y) = \psi_y \ln(\epsilon_{t-1}^y) + \varepsilon_t^y \tag{30}$$

where ψ_π measures the degree of persistence of the ϵ_t^y and ε_t^y is a price-elasticity shock with variance σ_y^2. In symmetrical equilibrium, the first-order conditions imply the Phillips curve nonlinear, given by:

$$\frac{\epsilon_t^y}{x_t} - \kappa_\pi\left(\pi_t - \pi_{t-1}^{\iota_p}\overline{\pi}^{1-\iota_p}\right)\pi_t + \beta E_t\left[\Lambda_{t,t+1}\kappa_\pi\left(\pi_{t+1} - \pi_t^{\iota_p}\overline{\pi}^{1-\iota_p}\right)\pi_{t+1}^2\frac{y_{t+1}}{y_t}\right] = \epsilon_t^y - 1 \tag{31}$$

where $\Lambda_{t,t+s} = \beta^s \frac{u'(c_{t+s})}{u'(c_t)}$ is the stochastic discount factor, since households own the retail

[6] The introduction of nominal price rigidity allows us to analyze the real effects of monetary policy shocks.

[7] The retailers' optimization problem is described in Appendix D.

firms, and $x_t = \frac{p_t(i)}{p_t^w(i)} = mc_t(i)$ is the mark-up of the the final good price.

3.5 Central Bank

Suppose a Taylor rule implements monetary policy with interest rate smoothing, which respond to both the deviation of the gross inflation rate from the inflation target $\overline{\pi}$ and the deviation of output from its steady-state $\overline{y}$. The Central Bank controls the gross nominal interest rate R_t^d on bank deposits and risk-free bonds, following the Taylor rule (32):

$$R_t^d = \rho_r R_{t-1}^d + (1-\rho_r)[R^{d,ss} + \phi_\pi(\pi_t - \overline{\pi}) + \phi_y(y_t - \overline{y})] + \varepsilon_t^R \tag{32}$$

where $R^{d,ss}, \overline{\pi} = \pi^{ss}$ and $\overline{y} = y^{ss}$ represent steady-state values, and ε_t^r is a monetary policy shock which is a white noise process with zero mean and variance σ_r^2. The coefficient $\rho_r \in [0,1]$ is the interest rate smoothing parameter, and $\phi_\pi \geq 0$ and $\phi_y \geq 0$ are feedback parameters that reflect the sensitivity of the interest rate R_t^d to inflation and output deviations. The policy rate R_t^d is a weighted average of the lagged nominal interest rate R_{t-1}^d and the current target rate $R^{d,ss}$, which depends positively on the deviation of inflation from its target $\overline{\pi}$ and the deviation of output from its steady-state value $\overline{y}$. Let R_t^{dr} denote the gross real interest rate, thus the relation between the nominal and real interest rates is given by the Fisher equation:

$$R_t^{d,r} = E_t\left[\frac{R_t^d}{\pi_{t+1}}\right] \tag{33}$$

3.6 Imperfect Banking Competition (Cournot)

The Cournot banking sector is used to characterize oligopolistic competition and capture banks' market power once the banking sector tends to be dominated by a few large players. In a Cournot equilibrium, banks' quantity-setting decisions affect the market loan rate. Assume there are N banks in the economy, indexed by j, which operate under Cournot competition. Each bank considers the effect of its choice $b_t(j)$ on the entrepreneurs' capital and loan demand through the equilibrium lending rate but ignores the general equilibrium effects and takes other aggregate prices and quantities as indicated.

The banks' activity helps finance operations carried out by entrepreneurs, such as the purchase of capital and payment of salaries. To this end, in Li (2019), banks finance loans from deposits obtained from households and return an interest rate to them. The banks also return all of their profits to the households at the end of the period. In our model, banks do not return all of their profits to households, but only a fraction of their total profits in the form of dividends and accumulate the remainder in the form of bank capital.[8] This banks' behavior also differs from Gerali et al. (2010), where banks have a zero dividend policy and pay a cost of managing their capital.[9]

[8]We can think of households as bank shareholders.

[9]The bank's capital can be considered the bank's equity.

The capital accumulated by banks can be used together with the deposits collected to finance new loans for entrepreneurs. Then, the banks have the following balance-sheet identity:

$$b_t(j) = d_t(j) + k_t^B(j) \tag{34}$$

where $k_t^B(j)$ is the bank's capital, $d_t(j)$ are the deposits received from households and $b_t(j)$ are the loans made to the entrepreneurs in period t by the bank j. The banks' capital is accumulated out of retained earnings:

$$k_t^B(j) = (1 - \delta^b) k_{t-1}^B(j) + \Gamma_t^B(j) - div_t^B(j) \tag{35}$$

where $\Gamma_t^B(j)$ are overall real profits made by bank j, δ^B measures the resources used in managing capital, and div_t^B are the dividends paid to households in the period t. We also assume, in the same line of Gerali et al. (2010), that banks have an optimal exogenous target τ^B for their capital-to-loans ratio, deviations from which imply a quadratic cost. The optimal capital-to-loans ratio can be considered a simple shortcut to studying the implications and costs of regulatory capital requirements or capturing the tradeoffs that arise when banks need to decide how many resources to save or borrow. This cost related to the capital position of the bank j is given by $\Omega_t^B(j)$:

$$\Omega_t^B(j) = \frac{\kappa_{k^B}}{2} \left(\frac{k_t^B(j)}{b_t(j)} - \tau^B \right)^2 k_t^B(j) \tag{36}$$

where the bank j pays a quadratic cost parameterized by a coefficient κ_{k^B} whenever the capital-to-loans ratio $\frac{k_t^b(j)}{b_t(j)}$ deviates from the optimal target value τ^B. Then, the bank j profit in the interbank market organized under Cournot competition in period t is:

$$\Gamma_t^B(j) = \frac{1}{\pi_t} \left[R_{t-1}^b \left(b_{t-1}(j) + \sum_{m \neq j} b_{t-1}(m) \right) b_{t-1}(j) - R_{t-1}^d d_{t-1}(j) - \Omega_{t-1}^B(j) \right] \tag{37}$$

which R_t^b is the nominal interest rate paid by entrepreneurs for loans $b_t(j)$ taken from the bank j, R_t^d is the nominal interest rate determined by the Central Bank paid on household's deposits, and $b_t(m)$ are loans granted by banks $m \neq j$ in the interbank market. In the imperfect competition environment, R_t^b represents the inverse of the loan demand function which dependes on b_t and therefore of $b_t(j)$. The dependence of R_t^b on $b_t(j)$ means that each bank j has a certain control over the equilibrium gross loan interest rate R_t^b by changing its own quantity of loans $b_t(j)$ given the other quantity of loans $b_t(m)$ granted by banks $m \neq j$ in the interbank market with Cournot structure. Thus, replacing the balance-sheet identity (34) in the equation (37):

$$\Gamma_t^B(j) = \frac{1}{\pi_t} \left[R_{t-1}^b \left(b_{t-1}(j) + \sum_{m \neq j} b_{t-1}(m) \right) b_{t-1}(j) - R_{t-1}^d (b_{t-1}(j) - k_{t-1}^B(j)) - \Omega_{t-1}^B(j) \right] \tag{38}$$

Note that the specification of the banks' profits (38) implies that yesterday's decisions determine today's profits. This reflects the inherently intertemporal nature of finance, but the timing choice does not materially change the optimization and forward-looking bankers' problem in the absence of credit risk. Then, it is possible to define capital accumulation $k_t^B(j)$ as follows (replacing (38) in (35)):

$$k_t^B(j) = \left(1 + \frac{R_{t-1}^d}{\pi_t} - \delta^b\right) \frac{k_{t-1}^B(j)}{s_t^{k^B}} + \left(\frac{R_{t-1}^b - R_{t-1}^d}{\pi_t}\right) b_{t-1}(j) - div_t^B(j) - \frac{\Omega_{t-1}^B(j)}{\pi_t} \quad (39)$$

The bank capital accumulation (39) is subject to a financial shock $s_t^{k^B}$ that follows an autorregressive AR(1):

$$\ln(s_t^{k^B}) = \psi_{s^{k^B}} \ln(s_{t-1}^{k^B}) + \varepsilon_t^{sk^B} \quad (40)$$

where $\psi_{s^{k^B}}$ measures the degree of persistence of $s_t^{k^B}$ and $\varepsilon_t^{sk^B}$ is a financial shock that destroys the capital accumulated by banks with variance $\sigma_{s^{k^B}}^2$. Therefore, each bank j maximizes the sum of the present discounted value of future dividends subject to bank capital accumulation law (39):[10]

$$\max_{\{b_t(j), k_t^b(j), div_t^B(j),\}} E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} [\ln(div_{t+s}^B(j))] \quad (41)$$

where $\Lambda_{t,t+s} = \beta^s \frac{u'(c_{t+s})}{u'(c_t)}$ is the stochastic discount factor, since households own the banks. Solving the banks' optimization problem with respect to $b_t(j)$ gives the following first-order condition:

$$E_t \Lambda_{t,t+1} \left\{ \frac{\lambda_{t+1}^B(j)}{\pi_{t+1}} \left[\frac{\partial \Omega_t^B(j)}{\partial b_t(j)} - \left(\frac{\partial R_t^b}{\partial b_t(j)} b_t(j) + R_t^b - R_t^d \right) \right] \right\} = 0 \quad (42)$$

In a Cournot equilibrium, the total optimal loan quantity is $b_t = b_t(j) + \sum_{m \neq j} b_t(m)$ and each bank produces a share of the total quantity of loans b_t. Besides that, the total optimal bank capital is $k_t^B = k_t^B(j) + \sum_{m \neq j} k_t^B(m)$ and each bank accumulates a share of the total bank capital in the interbank market k_t^B. Assuming banks are identical, then $b(j) = \frac{b_t}{N}$ and $k_t^B(j) = \frac{k_t^B}{N}$ in equilibrium. Since $\frac{\partial R_t^b}{\partial b_t(j)} = \frac{\partial R_t^b}{\partial b_t} \frac{\partial b_t}{\partial b_t(j)} = \frac{\partial R_t^b}{\partial b_t}$ in Cournot equilibrium, the first-order condition (42) can be written as:

$$E_t \Lambda_{t,t+1} \left\{ \frac{\lambda_{t+1}^B}{\pi_{t+1}} \left[\frac{\partial \Omega_t^B}{\partial b_t} - \left(\frac{\partial R_t^b}{\partial b_t} \frac{b_t}{N} + R_t^b - R_t^d \right) \right] \right\} = 0 \quad (43)$$

where the market loan demand is given by entrepreneurs' binding borrowing constraint (11). The loan rate R_t^b has a direct negative effect on market loan demand b_t since an increase in R_t^b reduces the entrepreneurs' borrowing capacity. Besides, R_t^b also has an indirect effect on b_t by influencing the entrepreuners' demand for physical capital k_t.[11]

[10] The banks' optimization problem is described in Appendix E.
[11] It can be seen by the equation (16).

When bank j chooses $b_t(j)$ to maximize dividends, it needs to consider how entrepreneurs would respond by changing their demand for physical capital $\frac{\partial k_t}{\partial R_t^b}$, what affects the level of investments in the economy.

The entrepreneurs' demand for capital k_t decreases in the loan rate R_t^b because $\frac{\partial k_t}{\partial R_t^b} < 0$ and the interest rate elasticity of capital demand $PEK_t \equiv -\frac{\partial k_t}{\partial R_t^b}\frac{R_t^b}{k_t}$ monotonically decreases in the expected marginal product of capital:

$$PEK_t = \frac{1}{1-\alpha}\left(\frac{m_t^k E_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b}\right]}{E_t\left[\Lambda_{t,t+1}^E MPK_{t+1}\right]}\right) \tag{44}$$

where $MPK_{t+1} \equiv \frac{\alpha z_{t+1}(k_t)^{\alpha-1}(l_{t+1})^{1-\alpha}}{x_{t+1}}$ is the marginal product of capital in real terms. The market loan demand elasticity PED_t captures their dependency on the capital demand elasticity PEK_t. The elasticity of entrepreneurs' loan demand concerning equilibrium gross loan rate R_t^b under Cournot competition is:[12]

$$PED_t \equiv -\frac{\partial b_t}{\partial R_t^b}\frac{R_t^b}{b_t} = 1 + PEK > 0 \tag{45}$$

Solving the first order condition (43), it is possible to find the following expression for the loan interest rate R_t^b, with $\Lambda_{t,t+1} > 0$ and $\pi_{t+1} \equiv \frac{p_{t+1}}{p_t} > 0$:

$$R_t^b = \frac{R_t^d - \kappa_{k^B}\left(\frac{k_t^B}{b_t} - \tau^B\right)\left(\frac{k_t^B}{b_t}\right)^2}{\left(1 - PED_t^{-1}\frac{1}{N}\right)} \tag{46}$$

where N is the number of banks' and κ_{k^B} is the banks capitalization cost. From equation (46), the loan interest rate R_t^b decreases in the number of banks N (more banking competition) and in the loan demand elasticity PED_t, entrepreneurs respond quickly to increased loan interest rate R_t^b and reduce the amount of loans b_t demanded, forcing the banks to charge a lower loan interest rate. The capital-to-loans rate $\frac{k_t^B}{b_t}$ is always below of the target τ^B, so the condition $R^b > R^d$ is always valid.

The banks' optimization problem also returns the following condition for dividends:

$$div_t^B = \left(E_t\Lambda_{t,t+1}\left\{\frac{1}{div_{t+1}^B}\right\}\left\{(1-\delta^B) + \frac{1}{\pi_{t+1}}\left[R_t^d - \kappa_{k^B}\left(\frac{k_t^B}{b_t} - \tau^B\right)\left(\frac{3}{2}\left(\frac{k_t^B}{b_t}\right) - \frac{\tau^B}{2}\right)\right]\right\}\right)^{-1} \tag{47}$$

where $div_t^B = div_t^B(j) + \sum_{j\neq m} div_t^B(m)$ are the optimal total dividends paid for households in the economy. In the Cournot equilibrium, the optimal total dividens are $div_t^B = \frac{div_t^B(j)}{N}$. From (47), dividends paid in the period-t decreases in the amount of capital accumulated by banks k_t^B and increses in expected inflation π_{t+1}.

[12]See Appendix F.

3.7 Equilibrium Conditions

Implicitly, a equilibrium was imposed in the deposit and loan markets. By Walras' law, whether $n-1$ markets are in equilibrium, then n^{th} market is also in equilibrium. Thus, in equilibrium the aggregate resource constraint is:

$$c_t + i_t + \frac{\kappa_\pi}{2}(\pi_t - \overline{\pi})^2 y_t + \frac{\chi}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 i_t - div_t^B = y_t \tag{48}$$

which is also the goods market clearing condition. In equilibrium, the new capital supplied by capital producers is equals the firms' capital demand and the labor supplied by households equals the firms' labor demand. In addition, the equilibrium in the Cournot banking sector can be written as $b_t^B = \sum_{j=1}^{N} b_t(j)$, $d_t^B = \sum_{j=1}^{N} d_t(j)$ and $k_t^B = \sum_{j=1}^{N} k_t^B(j)$, where the supply of loans from the banking sector b_t^B is equals the market loan demand b_t, the demand for deposits from the banking sector d_t^B equals the supply of deposits from households d_t, and the interbank market banking capital k_t^B is equal to the sum of the N banks' capital. From (34), the total loan supply b_t^B equals the total deposit holding in the interbank market plus the total capital accumulated k_t^B in the banking sector, $b_t^B = d_t^B + k_t^B$.

4 Bayesian Estimation and Calibration

4.1 Data

Our model uses the quarterly time series of 6 Brazilian variables from 2000-Q3 to 2019-Q4 (77 observations). These observations correspond to most of the period that private banks showed a high accumulation of assets. We have chosen as variables: GDP, investment, IPCA inflation (official inflation index adopted in Brazil), nominal short-term interest rate (Selic), loans to firms (working capital) and deposits.[13]

The GDP, investment, and IPCA inflation data source are the Brazilian Institute of Geography and Statistics (IBGE).[14] The other data referents to the short-term interest rate (Selic), loans to firms and deposits were extracted from the Central Bank of Brazil (BCB).[15] The short-term interest rate (Selic) is the benchmark Brazilian interest rate used as a basis for setting other rates in the financial system. The Table 1 below summarizes the observable variables used in the estimation:

[13]In Appendix H there is a more detailed description of the data used in the estimation.

[14]IBGE data can be found on the website www.ibge.gov.br.

[15]BCB data can be found on the website www.bcb.gov.br.

Table 1: Description of the observable variables used in the estimation

Variables	Series	Source
y_t^{obs}	GDP - seasonally adjusted	IBGE
i_t^{obs}	Investiment - seasonally adjusted	IBGE
π_t^{obs}	IPCA index - CPI inflation	IBGE
R_t^{obs}	Short-term nominal interest rate - Selic annualized	BCB
b_t^{obs}	Credit operations with nonearmarked funds (end of period)	BCB
d_t^{obs}	Deposit money banks - Time deposits, savings and others	BCB

4.2 Estimation Methodology

In this section, we will show the techniques used to estimate the parameters of the model. Our model is estimated using full-information likelihood-based Bayesian methods following An and Schorfheide (2007). The choice of this Bayesian estimation technique considered the analysis of several authors on the best estimation techniques. Rabanal and Rubio-Ramírez (2005) argue that the Bayesian approach can estimate the entire DSGE model, unlike the GMM method, which is based on a particular equilibrium relationship. Another argument the authors use is that the Bayesian technique is more efficient for small samples than GMM. In addition, the Bayesian approach allows the use of prior distributions that function as weights in the posterior distribution process.

Herbst and Schorfheide (2015) emphasize that the use of a priori distributions facilitates the process of identifying the model's parameters and minimizes the problem of extreme values, that is, coefficient values that do not reflect the reality of the economy represented in the model. In summary, a Bayesian method uses the priors of the parameters $p(\theta)$ combined with the likelihood of the DSGE model $L(Y|\theta)$ to produce the posterior joint distribution of the parameter vector $p(\theta|Y)$, where $Y = \{y_1, \cdots, y_n\}$. The likelihood function is used to update the prior beliefs about the parameters conditioned on the sample information.

We also adopted a Monte-Carlo Markov-Chain (MCMC) sampling algorithm to simulate the parameter vector θ distribution since the posterior distributions are difficult to characterize. Then, the random-walk Metropolis-Hastings algorithm, which belongs to the class of MCMC algorithms, is used to generate a sample to have an approximation (draws) of the posteriors distributions. In the MCMC method, the objective is to obtain a sample of the posterior distribution and calculate sample estimates of distribution's characteristics through iterative simulation techniques based on Markov chains. The idea is to simulate a random walk in the parameter space θ that converges to a stationary distribution of interest for the estimation.

4.3 Calibration

In addition to estimation, calibration of some parameters such as steady-state values and parameters weakly identified by the observed variables is necessary to get the best forecasts from the model. The households subjective discount factor β and the entrepreneurs subjective discount factor β^E, following De Castro et al. (2015) and Gerali et al. (2010), are equal to $\beta = 0.989$ and $\beta^E = 0.97$. The capital share α and depreciation rate of physical capital δ are calibrated with the values 0.44 and 0.025, respectively, according to Li (2019). The relative utility weight on leisure time ϕ_l is set to 1.8 to yield a steady-state labor l^{ss} around 0.25 (people work for around 7 hours a day, on average). The calibration for ϵ, elasticity of substitution among retail goods, is in line with the Gerali et al. (2010) and Andrés and Arce (2012), and was defined as $\epsilon = 6$ to generate a 20% mark-up $\left(x = \frac{\epsilon}{\epsilon-1}\right)$ over differentiated goods in the zero-inflation steady-state and the gross inflation target is $\overline{\pi} = 1$.

Given the calibration for $\beta, \beta^E, \alpha, \delta$ and ϵ, to study the effects of imperfect banking competition, we defined the number of banks $N = 5$. The target for the capital-to-loans ratio τ^B is equal to 0.17 and the costs for bank capital management δ^B is equal to 0.0944, following da Silva et al. (2012). The calibration of the parameters can also be seen in the Table 2 below:

Table 2: Calibrated parameters

Parameter	Value	Description
Households		
β	0.989	Subjective discount factor
ϕ_l	1.8	Relative utility weight on leisure time
entrepreneurs		
β^E	0.97	Subjective discount factor
α	0.44	Physical capital share
δ	0.025	Depreciation rate for physical capital
Retail firms		
ϵ	6	Elasticity of substitution between retail goods
Banking sector		
N	5	Number of banks
δ^B	0.0944	Cost of managing the position of bank capital
τ^B	0.17	Target for the capital-to-loans ratio

4.4 Prior Distribution

We used previous studies for prior information and followed the specifications cited in Gerali et al. (2010) and da Silva et al. (2012). The prior for the investment adjustment costs χ is set at the value used by Gerali et al. (2010), is assumed to follow a gamma distribution with a mean of 2.5 and with a standard deviation of 1.0. The gamma distribution is also used as a prior for the banks' capitalization cost κ_{k^B} and the price adjustment cost of retail firms κ_π, according to Gerali et al. (2010). We defined κ_{k^B} with a mean 10.0 and standard deviation of 5.0 and set the κ_π with a mean 50.0 and standard deviation of

20.0. The price indexation parameter ι_p was defined with a mean of 0.65 and a standard deviation of 0.20.

The normal and gamma distributions are used as main priors for monetary policy rule parameters in the interest rate rule R^d. The exception is the parameter determining the degree of interest rate smoothing ρ_r, which the literature uses the beta distribution. The prior mean for the ρ_r is 0.75, and the standard deviation of 0.10. The coefficient for the output gap ϕ_y follows a normal distribution with a mean of 0.10 and a standard deviation of 0.15. Lastly, the coefficient for the response to inflation ϕ_π has a prior mean equal to 2.0 and a standard deviation of 0.50 with gamma distribution. All prior distributions mentioned in the monetary policy rule were defined according to Gerali et al. (2010).

The prior means for the all auto-regressive coefficients $(\psi_z, \psi_k, \psi_{qk}, \psi_\pi, \psi_{k^B})$ were set to 0.80, with standard deviations of 0.10. For these coefficients we also follow the Gerali et al. (2010) and use the beta distribution as the prior distribution. The priors' means of the shocks $(\sigma_z, \sigma_k, \sigma_{qk}, \sigma_\pi, \sigma_{k^B}, \sigma_R)$ are assumed to follow inverse-gamma distribution with value 0.01 and standard deviation of 0.05 as da Silva et al. (2012). The prior distributions chosen for the estimated parameters can be seen in the Table 3.

4.5 Posterior Results

Table 3 report the results of estimates that summarize the means and the 10th and 90th percentiles of the posterior distributions.[16] The posterior mean estimated from Brazil data for the adjustment cost of prices $\kappa_\pi = 86.35$ and investment adjustment cost $\chi = 5.02$ are higher that their prior and both are high compared to the estimated values for the European Union and see in Gerali et al. (2010), $\kappa_\pi = 30.57$, and Li (2019), $\chi = 2.50$. Regarding the cost of capitalization of banks $\kappa_{k^B} = 22.19$, we found a higher value for Brazil than that found for Europe $\kappa_{k^B} = 11.49$, in Gerali et al. (2010). This high value of the cost of capitalization of Brazilian banks means that any shock that affects the capital-to-loans ratio $\frac{k_t^B}{b_t}$ has the power to amplify the spread variation.

About the estimated parameters of the Taylor rule, the interest rate smoothing is estimated in $\rho_r = 0.62$, the response to the deviation of inflation from the target has a posterior mean equal to $\phi_\pi = 1.56$, and the response to the output gap is equal to $\phi_y = 0.33$ higher than in the prior. The estimate of the price indexation parameter found a value equal to $\iota_p = 0.73$. Estimates of autoregressive coefficients show that some shocks have high persistence, such as productivity, bank capital, and output. All these parameters have a higher posterior mean than the previous one. On the other hand, collateral and investment shocks have a lower persistence since autoregressive parameters have a posterior mean lower than the prior.

[16] Appendix H provides the graphs of the priors and posteriors of the structural parameters.

Table 3: Prior and posterior distribution of structural parameters

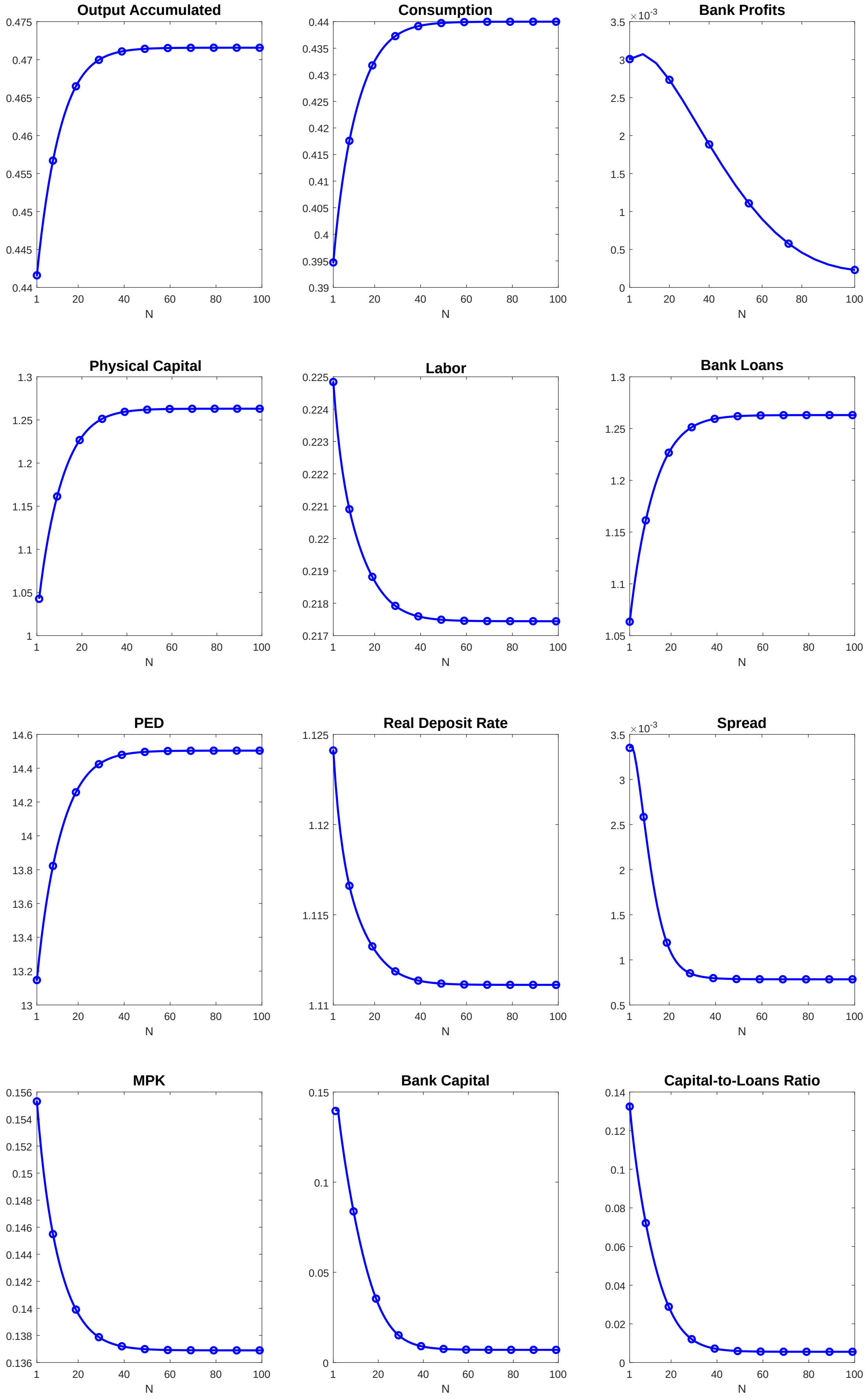
Parameter description			Prior			Posterior		
			Distribution	Mean	Std. dev	Mean	90%HP	
κ_π	Adjustment Cost	prices	gamma	50.0	20.0	86.35	35.28	98.29
χ		investiment	gamma	2.5	1.0	5.020	2.987	6.906
κ_{k^B}		bank capital	gamma	10.0	5.0	22.19	17.15	27.04
ρ_R	Taylor Rule	persistence	beta	0.75	0.10	0.620	0.501	0.736
ϕ_π		inflation	gamma	2.0	0.5	1.565	0.759	2.303
ϕ_y		output	normal	0.10	0.15	0.336	0.220	0.451
ι_p	Indexation	prices	beta	0.65	0.20	0.735	0.509	0.985
ψ_z	AR process	productivity	beta	0.80	0.10	0.919	0.875	0.965
ψ_k		collateral	beta	0.80	0.10	0.391	0.265	0.519
ψ_{qk}		investiment	beta	0.80	0.10	0.404	0.268	0.547
ψ_y		output	beta	0.80	0.10	0.795	0.648	0.960
ψ_{k^B}		bank capital	beta	0.80	0.10	0.903	0.873	0.932
σ_z	Shocks	productivity	inv. gamma	0.01	0.05	0.035	0.029	0.040
σ_k		collateral	inv. gamma	0.01	0.05	0.050	0.043	0.058
σ_{qk}		investiment	inv. gamma	0.01	0.05	0.061	0.050	0.072
σ_y		output	inv. gamma	0.01	0.05	0.082	0.024	0.146
σ_R		policy rate	inv. gamma	0.01	0.05	0.064	0.046	0.081
σ_{k^B}		bank capital	inv. gamma	0.01	0.05	0.097	0.085	0.114

Finally, about the standard deviation estimates, the negative shocks for productivity, collateral, bank capital, and investment aim to verify the influence of these shocks on the spread, and consequently, on the real variables of the economy.

5 Comparative Static with Number of Banks N

Figure 2 shows the steady-state values of essential variables in the model change with the number of banks N varying in the range of 1 to 100 in the absence of any shock. Higher N means more intense competition in the banking sector. When the number of banks N increases, the output y and consumption c increases, and the spread $(R^b - R^d)$ decreases. Unlike Li (2019), when banks accumulate capital k^B in an interbank market organized under Cournot competition, the spread does not become zero. It happens because the loan rate R^b (46) charged by banks on loans b will not be equal to rate R^d paid on deposits d due to the existence of the banks' capitalization cost $(\kappa_{k^B} \neq 0)$. Whether $N \to \infty$, banks can optimize by setting its capital-to-loans ratio $\frac{k^B}{b}$ less than its optimal target value τ^B such that $R^b > R^d$ remains valid.

Figure 2: Steady-state values for $N \in [1, 100]$



Note: The spread is expressed in percent points and MPK is calculated as $\alpha z k^{\alpha-1} l^{1-\alpha}$.

From (37), a lower loan interest rate decreases the profits collected by the banks, which reduces the accumulated banking capital (35). Once the number of banks N increases and physical capital k is also financed by bank loans, a lower loan rate makes physical capital cheaper relative to labor l for the entrepreneurs, increasing the capital-to-labor ratio $\frac{k}{l}$ and then reducing the marginal product of capital $\alpha z \left(\frac{k}{l}\right)^{\alpha-1}$.[17] From (44), capital demand elasticity PEK is more elastic when the MPK is lower and, consequently, it directly affects the loan demand elasticity, making the PED higher and reducing the spread. Since the bank loans are cheaper due to the lower loan rate, entrepreneurs will increase their leverage rate and, consequently, its production y. With a higher level of bank loans b in the economy and less capital accumulation by banks k^B, the capital-to-loans ratio $\frac{k^B}{b}$ decreases. This scenario could generate an effect of increasing the spread, which does not happen due to the opposite and higher impact of perfect banking competition that reduces the spread.

6 Dynamic Analysis

In this section, we investigate how the economy responds to negatives shocks: (i) productivity, (ii) financial, (iii) collateral, and (iv) investment. We assume two scenarios of imperfect banking competition (IBC). First, banks accumulate capital ($k^B \neq 0$). Second, banks do not accumulate capital ($k^B = 0$).

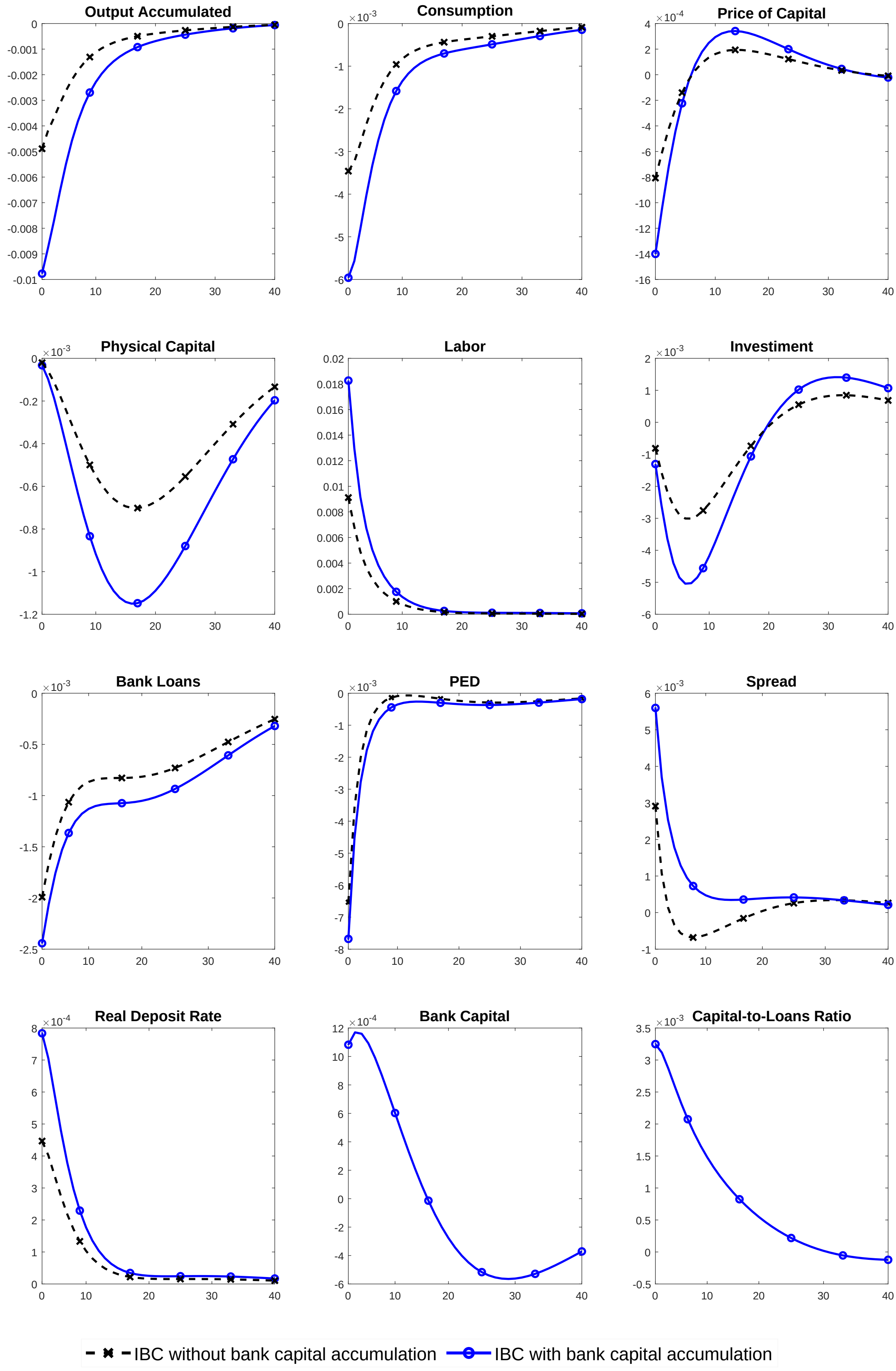
6.1 Productivity Shock

Figure 3 shows impulse responses after a persistent negative productivity shock z. Under the imperfect banking competition (IBC) with the accumulation of bank capital, the responses of output, investment, physical capital, and consumption are amplified to the scenario that banks do not accumulate capital. The amplification effect of real variables can be explained by the countercyclical spread and the cost of capitalization of the banking sector.

The rise in the spread is due to the combined effect of the bank's market power and the fall in the loan demand elasticity (PED) after the negative productivity shock that affects the entrepreneurs' asset prices q. The negative productivity shock reduces entrepreneurs' demand for physical capital and consequently causes a drop in their price q. The fall in the price of physical capital reduces entrepreneurs' borrowing capacity (11) and makes them more financially constrained. Then, the result is a more inelastic PED, as can also be seen in Figure 3. With imperfect banking competition, banks have market power and take advantage of the lower PED by reducing their loans quantities b to achieve a higher equilibrium loan rate R^b.

[17] Since $\alpha < 1$ under the assumption of a constant-returns-to-scale production function, marginal product of capital MPK decreases in the capital-to-labor ratio.

Figure 3: Impulse responses to a negative productivity shock



Note: The horizontal axis shows quarters after a negative productivity shock z at the beginning of period one. The vertical axis shows the percentage deviation from the steady-state for real variables, interest rates and the spread in percentage points.

The existence of capitalization cost κ_{k^B} explains the difference between the spreads when banks do not accumulate and accumulate capital because this cost κ_{k^B} interferes in the equilibrium loan rate R^b charged on loans to entrepreneurs. Banks always keep the capital-to-loans ratio $\frac{k^B}{b}$ below optimal τ^B exogenously defined by the Central Bank. Whenever shocks affect the capital-to-loans ratio, the effects of these shocks are transmitted to the loan rate and, consequently, the bank spread.

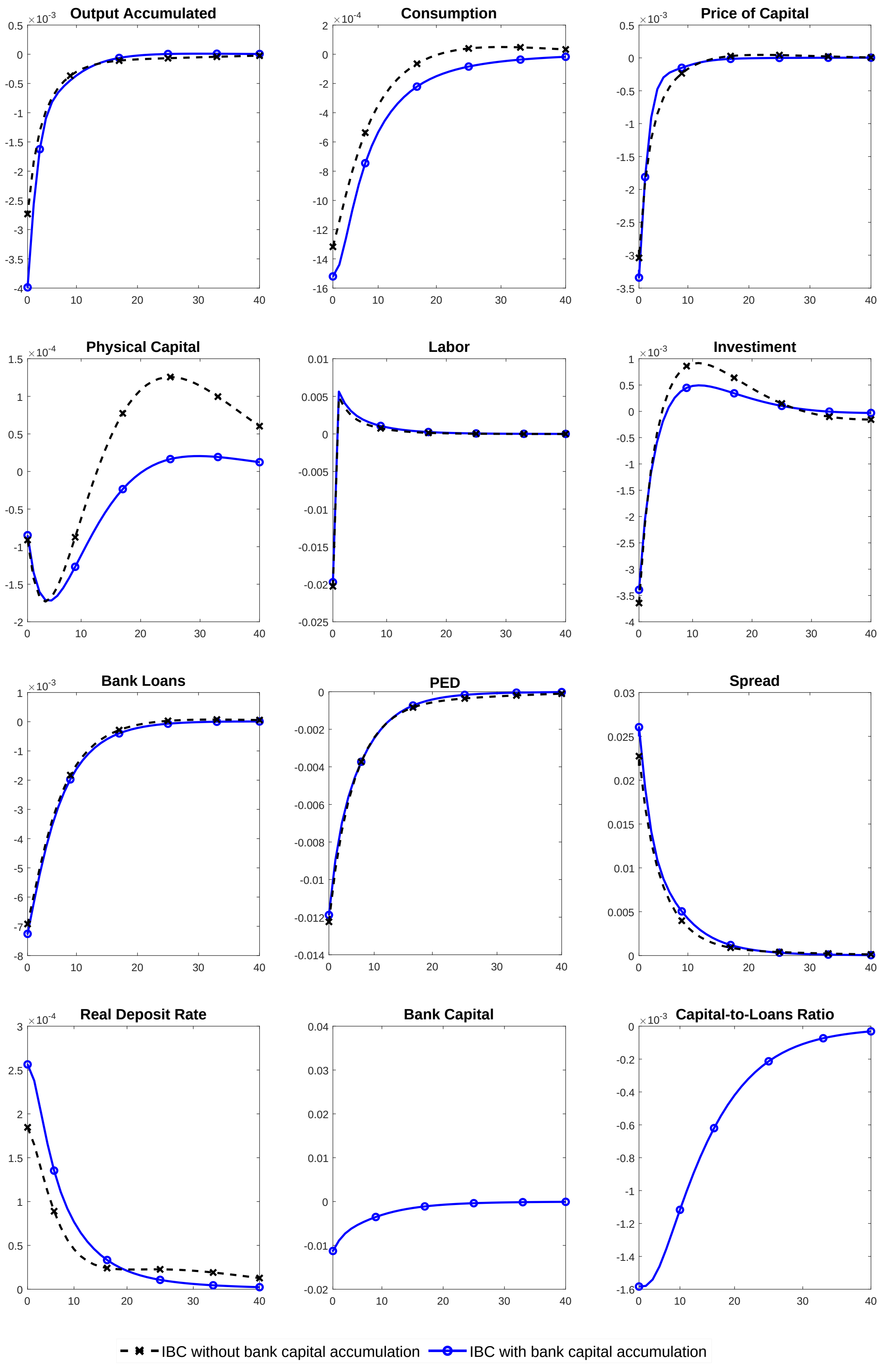
The banks' movement to reduce the number of loans b offered due to a negative productivity shock increases the capital-to-loans ratio from two actions. The first is naturally the drop in b, and the second is the increase in bank capital accumulated k^B due to the higher loan rate charged to financially constrained entrepreneurs. Even granting smaller loans, banks get more capital when the economy suffers a productivity shock that affects the price of entrepreneurs' assets. The spread increased in greater magnitude in the scenario that banks accumulate capital because of the movement made by the capital-to-loans ratio and transmitted to the loan rate R^b through the cost of capitalization κ_{K^B}. The decapitalization of entrepreneurs is not immediate but becomes more intense approximately 20 quarters after the negative productivity shock. Labor becomes the substitute for the fall in physical capital, and the reduction in the accumulated output is double the scenario in which banks do not get capital.

6.2 Collateral Shock

This section investigates the negative shock in the pledgeability ratio m^k, which can be interpreted as a collateral shock (Figure 4). This collateral shock is a supply-side shock since it directly affects the supply of credit to the entrepreneurs and, therefore, the output accumulated. After a negative collateral shock m^k at the beginning of period 1, the responses of output and consumption have an amplifying effect compared to a scenario that banks do not accumulate capital. The reaction of physical capital becomes more accentuated from quarter ten onwards, showing a slower recovery. The late answer of the physical capital can be explained by the low-value parameter ψ_{m^k} estimated from Brazilian economic data indicating the persistence of the m^k shock dissipates more quickly through the business cycle.

The exogenous reduction in the pledgeability ratio m^k directly reduces the fraction of the physical capital used as collateral by entrepreneurs to get loans from banks and therefore lowers the entrepreneur's borrowing capacity through the binding collateral constraint (11). The decrease in the m^k makes the entrepreneurs more financially constrained and reduces the loan demand elasticity PED, implying an increase in the spread given the imperfect banking competition environment. The reduction in investment and output due to lower entrepreneurs' activity after the negative shock m^k causes a secondary inflationary effect since the supply of consumer goods will be smaller. This inflationary effect will reduce the real debt burden and improve the entrepreneurs' borrowing capacity.

Figure 4: Impulse responses to a negative collateral shock



Note: The horizontal axis shows quarters after a negative collateral shock m^k at the beginning of period one. The vertical axis shows the percentage deviation from the steady-state for real variables, interest rates and the spread in percentage points.

This debt-deflation effect is dominated by the impact of a lower pledgeability ratio m^k that directly reduces the entrepreneurs' borrowing capacity. It is possible to notice that the shock that hits the collateral of the entrepreneurs and makes them more financially restricted also affects the capital accumulated k^B by the banks. The reduced demand for loans after the negative shock m^k also reduces the banks' capital accumulation, even if banks cover a higher loan rate R^b. This reduction in bank capital accumulation causes a drop in the capital-to-loans ratio and generates a second spread growth increase in the scenario in which banks accumulate capital.

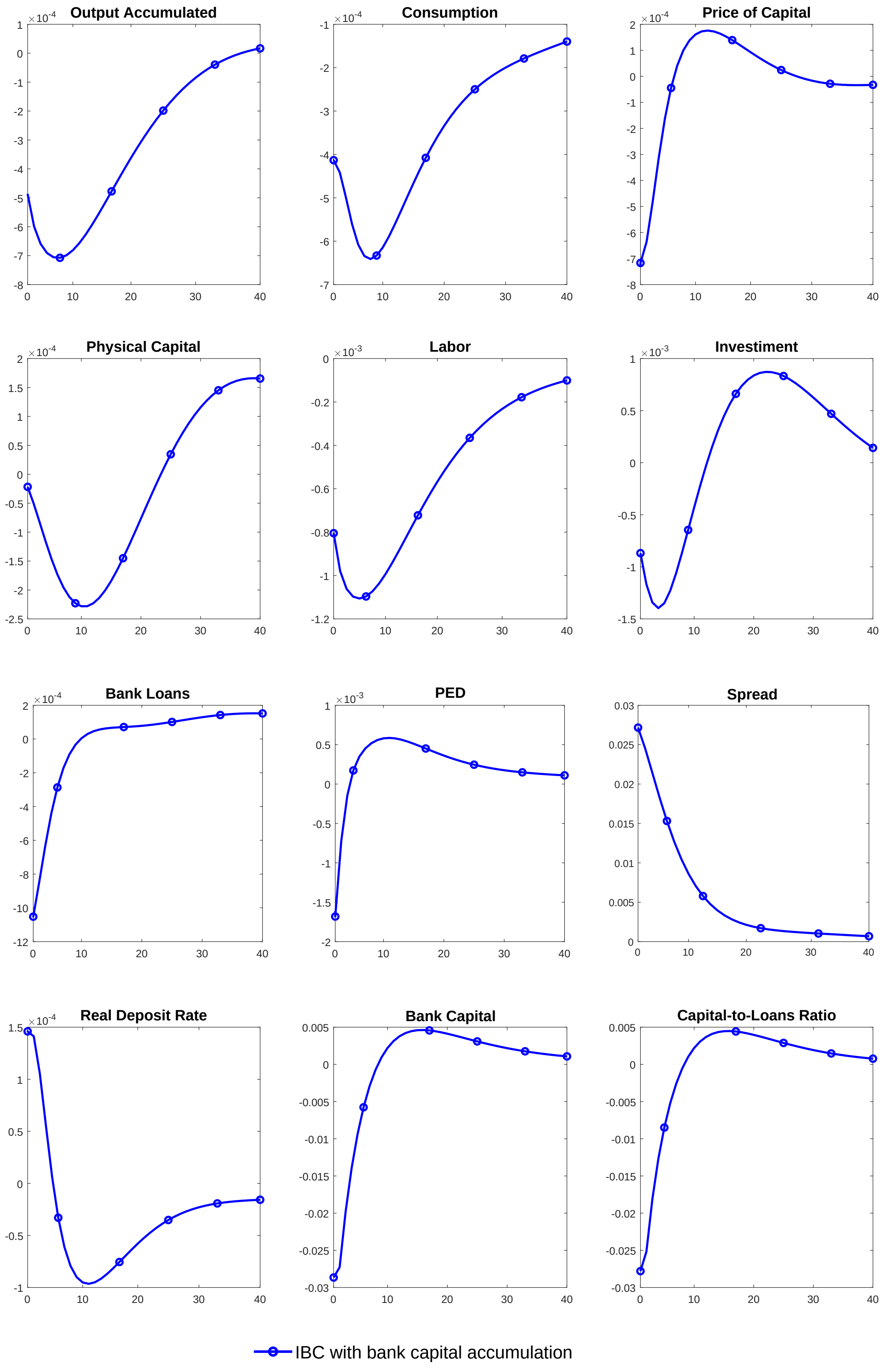
6.3 Financial Shock

To assess the countercyclical movement of the spread, it is also essential to recognize how the effects of shocks on banks' balance sheets affect the availability and cost of credit. As our model is particularly suited to analyzing this issue, in this section, we use it to study what happens if a bank's capital experiences a strong negative shock. To run the simulation, we introducing the possibility of an unexpected and persistent contraction of bank capital k^B. The negative financial shock has a strong persistence since the estimated parameter indicates a value equal to $\psi_{k^B} = 0.903$, greater than the prior. The financial shock destroys the bank's capital k^B and makes k^B fall more significantly than loans b. This process reduces the capital-to-loans ratio $\frac{k^B}{b}$ far from the target τ^B, requiring a fast recovery of banks. We also want to study the transmission and amplification mechanisms that account for the macroeconomic effects of bank capital losses in Figure 5.

The decline in bank capital k^B following the negative financial shock leaves banks too leveraged and with a burden of costs due to their deviation from capital requirements τ^B. So it is ideal for banks to rebalance assets and liabilities by reducing borrowing and, consequently, increasing the interest rate charged on loans. The reduction in loans b is a significant concern for banks as they need to approach the capital-to-loans target τ^B quickly. Loan volumes decrease for entrepreneurs, reducing the resources available to them. Entrepreneurs cut investment substantially because its relative cost increased as the loan rate R^b is higher due to the negative financial shock, and this increase the spread. The financial restriction of entrepreneurs due to the increase in the loan rate can also be seen in the fall in the PED. The fall in investment also affects labor, which has a negative deviation from the steady state. The lower demand for physical capital k also affects its price, causing a drop in q. With a lower level of investment, the accumulated output y of the economy falls, which is reflected in aggregate consumption c.

Banks balance their capital positions regardless of any external monetary stimulus. A rapid increase in the spread contributes to this rebalancing, rebuilding the bank's capital stock k^B. About the cost of deviating from the capital-to-loans ratio, the amplifying effect on the spread will be more significant if the κ_{k^B} value is high. It is possible to notice from

Figure 5: Impulse responses to a negative financial shock



Note: The horizontal axis shows quarters after a negative financial shock sk^B at the beginning of period one. The vertical axis shows the percentage deviation from the steady-state for real variables, interest rates and the spread in percentage points.

Figure 5 that after ten quarters of negative financial shock, the economy starts a recovery process. However, despite this change in the downward trajectory, the output takes about 40 quarters to return to the initial level. Aggregate consumption, on the other hand, has a recovery time of more than 40 quarters.

Entrepreneurs start their recovery some quarters before the entire economy. It is possible to notice that about 20 quarters after the negative financial shock, the PED returns to its initial value. The same recovery time can also be seen for the demand for physical capital k that accompanies entrepreneurs' recovery. This increase in demand for k raises the prices of capital q, which stabilize after 30 quarters after the financial shock. The labor already presents a slower recovery, taking more than 40 quarters for its complete recovery.

6.4 Investiment Shock

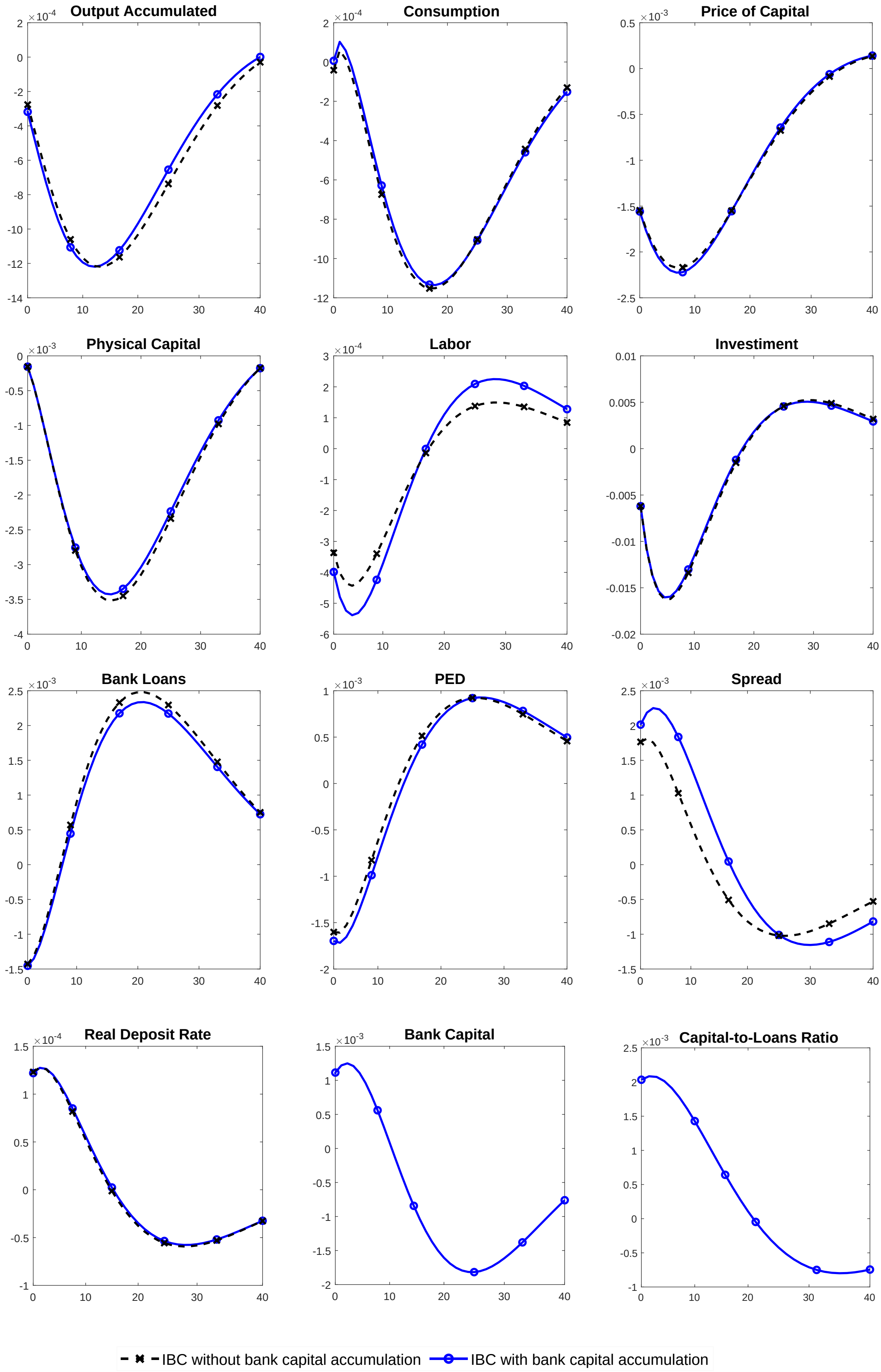
This section will investigate the effects of a negative shock that reduces the price of capital q and affects capital producers. The price of capital directly affects the entrepreneurs' PED and, consequently, their borrowing constraint. Entrepreneurs with inelastic PED provide the necessary incentive to increase the spread in an environment where banks have market power. The negative investment shock does not have a high persistence like productivity and financial shocks. The parameter that indicates persistence was estimated with a value equal to $\psi_{qk} = 0.404$ as can see in Table 3.

The effect of an exogenous fall in the price of capital q causes a fall in the PED of entrepreneurs. The banks, in response, reduce the number of loans b offered to obtain a higher loan rate. This mechanism naturally increases the spread in the interbank market. More financially constrained entrepreneurs to begin to decapitalize due to the reduction of loans granted by banks to operate business activities. The decapitalization process of entrepreneurs lasts about 20 quarters after the negative investment shock, and the total recovery takes more than 40 quarters. The drop in business activities also immediately affects the hired labor. The sharp decline in labor is reversed 20 quarters after the shock.

The increase in the bank spread allows banks to increase the accumulation of capital k^B, which, together with the fall in loans b, increases the capital-to-loans ratio. The difference between the spread charged when banks accumulate capital ($k^B \neq 0$) and in the scenario in which they do not ($k^B = 0$) is due to the cost of deviating from the target κ_{k^B} (or cost of bank capitalization). Figure 6 shows that though there is an increase in the capital-to-loans ratio, the difference about the target ($\tau^B - \frac{k^B}{b}$) is always positive, which generates an amplifying effect on the spread.

It is possible to notice that the spread amplifying effect dissipates after 20 quarters. The low persistence estimated for the shock from Brazilian data explains this rapid spread return to equilibrium. Shocks that pass through the price of capital, such as the productivity and investment shock, allow banks organized under imperfect competition to accumulate more capital as the spread increases. This immediate effect to increase the

Figure 6: Impulse responses to a negative investiment shock



Note: The horizontal axis shows quarters after a negative price of capital shock sq^k at the beginning of period one. The vertical axis shows the percentage deviation from the steady-state for real variables, interest rates and the spread in percentage points.

bank capital accumulation is not seen in shocks that hit the entrepreneurs' collateral and, naturally, in financial shocks that destroy bank capital.

In the collateral shock that reduces the number of entrepreneurs' assets, banks cannot accumulate more capital due to the drop in demand for loans despite charging a higher spread. And in the shock that destroys accumulated bank capital, banks take approximately 20 quarters to return to the initial equilibrium of bank capital. The accumulated output shows a persistent decline 15 quarters after the investment shock and needs more than 40 quartes to recover fully. The drop in consumption, in turn, is motivated by the reduction in investment and output levels, and it takes more than 40 quarters to return to the initial equilibrium.

7 Sensitive Analysis

This section checks the robustness of the baseline results in Section 6 by changing the investment adjustment cost parameter χ and the banks' capitalization cost κ_{k^B} when banks accumulate capital. We compare estimated values for Brazil in this paper with values used for the European Union in Li (2019) and Gerali et al. (2010), while all the other parameters are calibrated and estimated as in the baseline analysis. In a third step, we will test the robustness of the model by increasing the number of banks N in the interbank market and verifying the effects on the model's macroeconomic variables.

7.1 The Investment Adjustment Cost χ

After the estimations described in the Section 4 we found a value for the investment adjustment cost equal to $\chi = 5.02$ and higher than the value used by Li (2019) for the EU, $\chi = 2.5$. It is possible to notice that the increase in the investment cost χ, in the Figure 7, does not significantly affect the spread when we look at the productivity, financial and collateral shocks. The most significant amplifying impact on the spread is seen in the negative investment shock that affects precisely the capital producers responsible for χ. The difference in fall in the demand elasticity for loans PED is also more accentuated for the investment shock. The greater reduction in the price of capital q due to negative investment shock and the increase in χ causes a greater fall in the PED and a consequent greater amplifying effect on the spread.

The productivity shock and the investment shock make the capital-to-loans ratio increase. When the shock passes through the price of capital, making the PED inelastic, banks in the imperfect competition can immediately increase capital accumulation. In the presence of higher investment adjustment costs χ, imperfect banking competition significantly slows capital accumulation when banks accumulate capital. As a result, the drop in output is much more persistent for financial, collateral and investment shocks, taking about 40 quarters to reach a steady state. The productivity shock does not have an amplifying effect on the accumulated output when there is an increase in the adjustment cost, as this cost χ mainly affects the PED of the entrepreneurs.

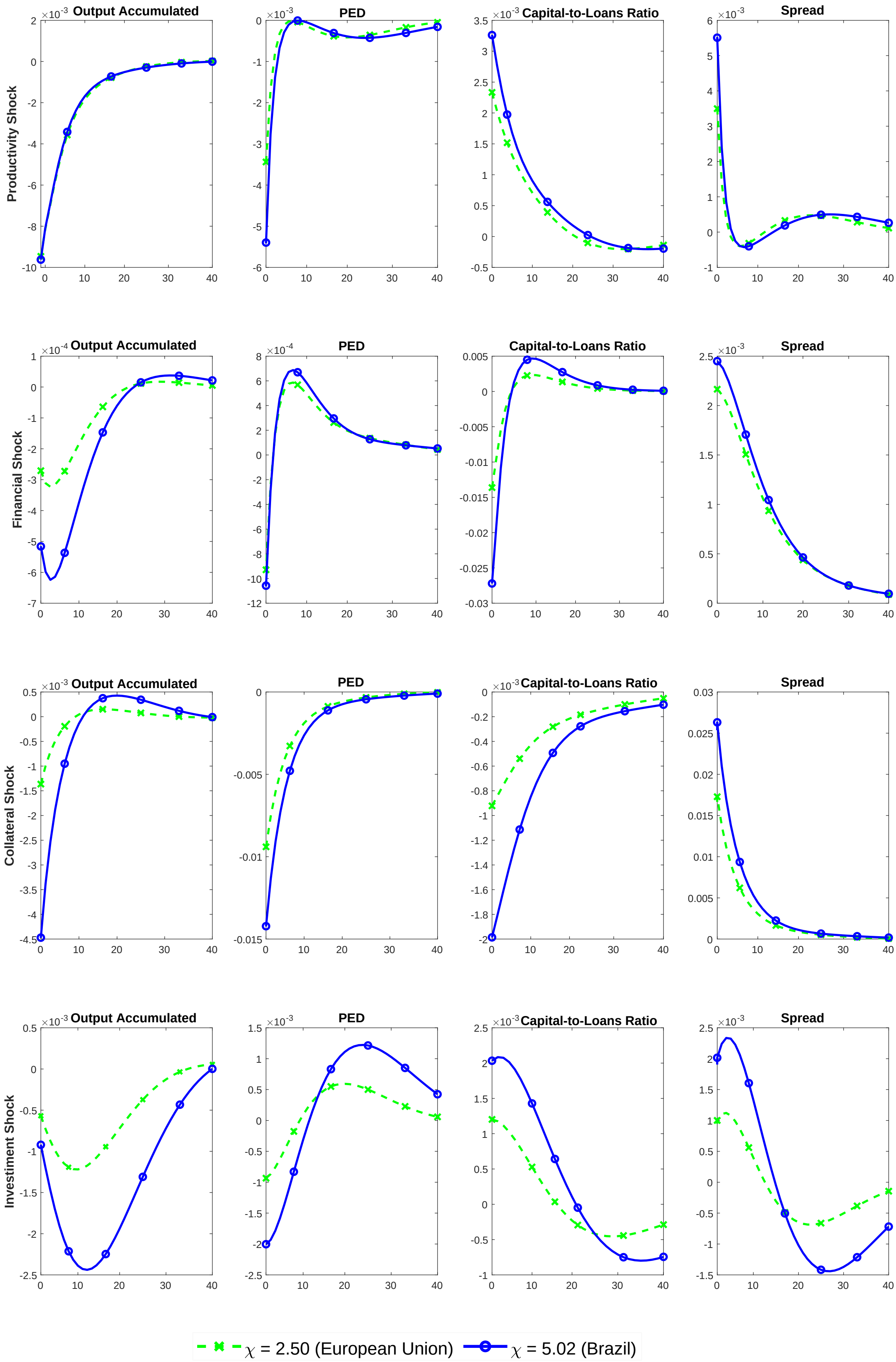
7.2 The Banks' Capitalization Cost κ_{k^B}

In Figure 8, we compare the effects of different shocks that hit the economy in a scenario that we vary the cost of capitalization of banks κ_{k^B}. The estimated cost of capitalization for Brazilian banks is equal to $\kappa_{k^B} = 22.19$ and is much higher than the cost found by Gerali et al. (2010) for the EU, $\kappa_{k^B} = 11.49$. This cost κ_{k^B} has the power to amplify the spread variation due to shocks that pass through the capital-to-loans ratio $\frac{k^B}{b}$.

In the financial shock that destroys the capital accumulated k^B by banks, there is an immediate drop in the capital-to-loans ratio, which significantly increases the spread due to the higher cost κ_{k^B}. The higher rate charged on loans makes entrepreneurs more financially constrained and inelastic PED, causing a secondary effect of increasing the spread. The accumulated output of the economy is also affected by the higher κ_{k^B} cost. The fall in y resulting from a financial shock tends to be amplified the greater the value of κ_{k^B}. The amplifying effect on the spread is also seen for the collateral shock that destroys entrepreneurs' assets. With less capital to use as collateral, entrepreneurs have a lower PED, which encourages banks with market power to charge a higher spread. This reduction in loan demand prevents banks from capitalizing, even setting a higher loan rate, bank capital accumulation falls and reduces the capital-to-loans ratio. The drop in the capital-to-loans ratio combined with the high κ_{k^B} cost significantly increases the spread.

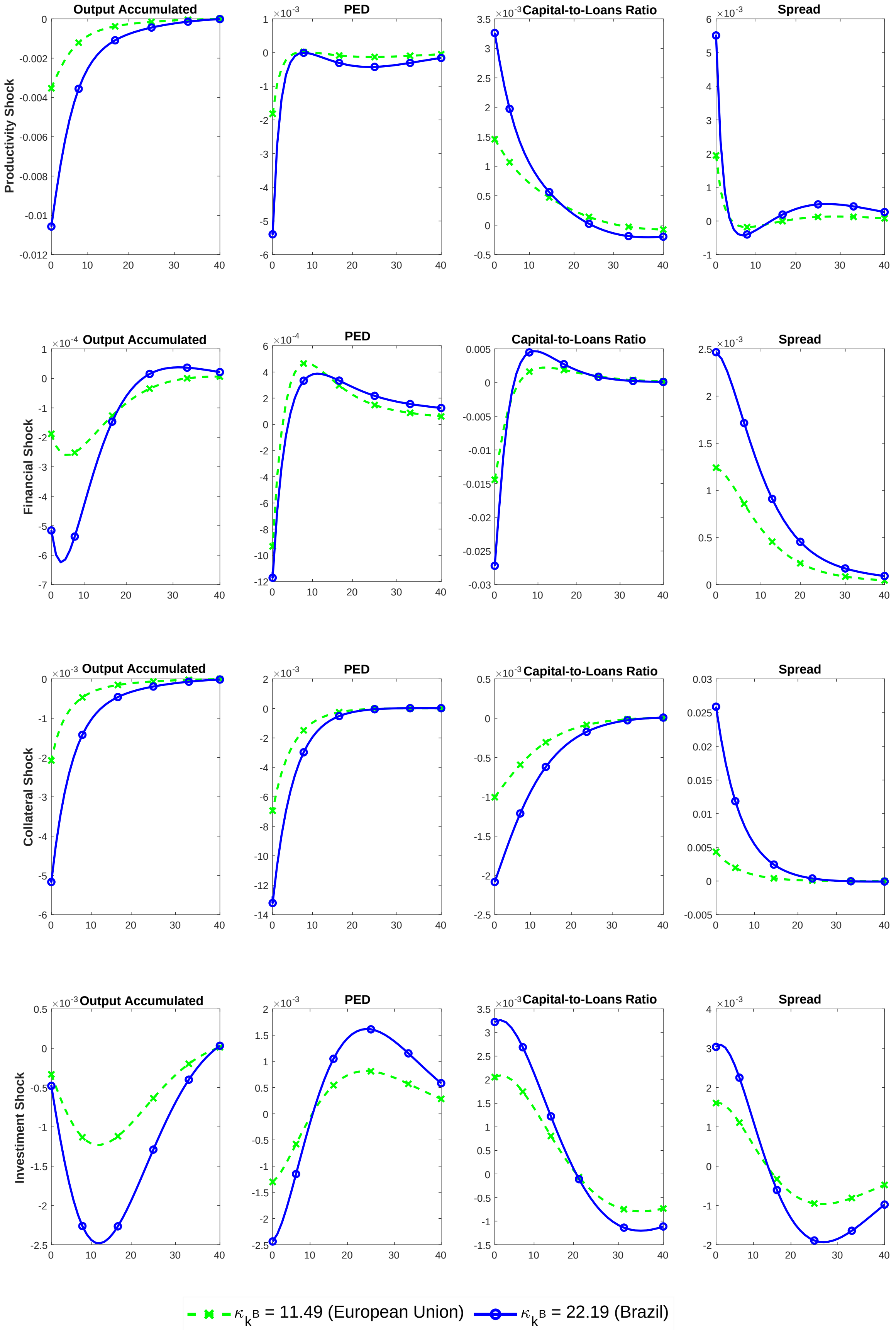
Regarding the productivity shock, the increase in κ_{k^B} does not significantly affect the spread variation. The trajectory of the spread returns to initial equilibrium after ten quarters. The accumulation of bank capital and reduced loans to entrepreneurs increases the capital-to-loans ratio. This movement contributes to the rise in the spread in a lesser magnitude as the capital-to-loans ratio is consistently below the target τ^B. For productivity, collateral, and investment shocks, the economy's accumulated output recovers on average 40 quarters after the adverse shocks. As for the financial shock, the return to the initial balance occurs after about 30 quarters.

Figure 7: Impulse responses to a negatives shocks with differents χ



Note: The horizontal axis shows quarters after a negatives shocks at the beginning of period one. The vertical axis shows the percentage deviation from the steady-state for real variables and the spread, which are expressed in deviations from the steady-state in percent points.

Figure 8: Impulse responses to a negatives shocks with differents κ_{k^B}



Note: The horizontal axis shows quarters after a negatives shocks at the beginning of period one. The vertical axis shows the percentage deviation from the steady-state for real variables and the spread, which are expressed in deviations from the steady-state in percent points.

7.3 The Effects of Banking Competition

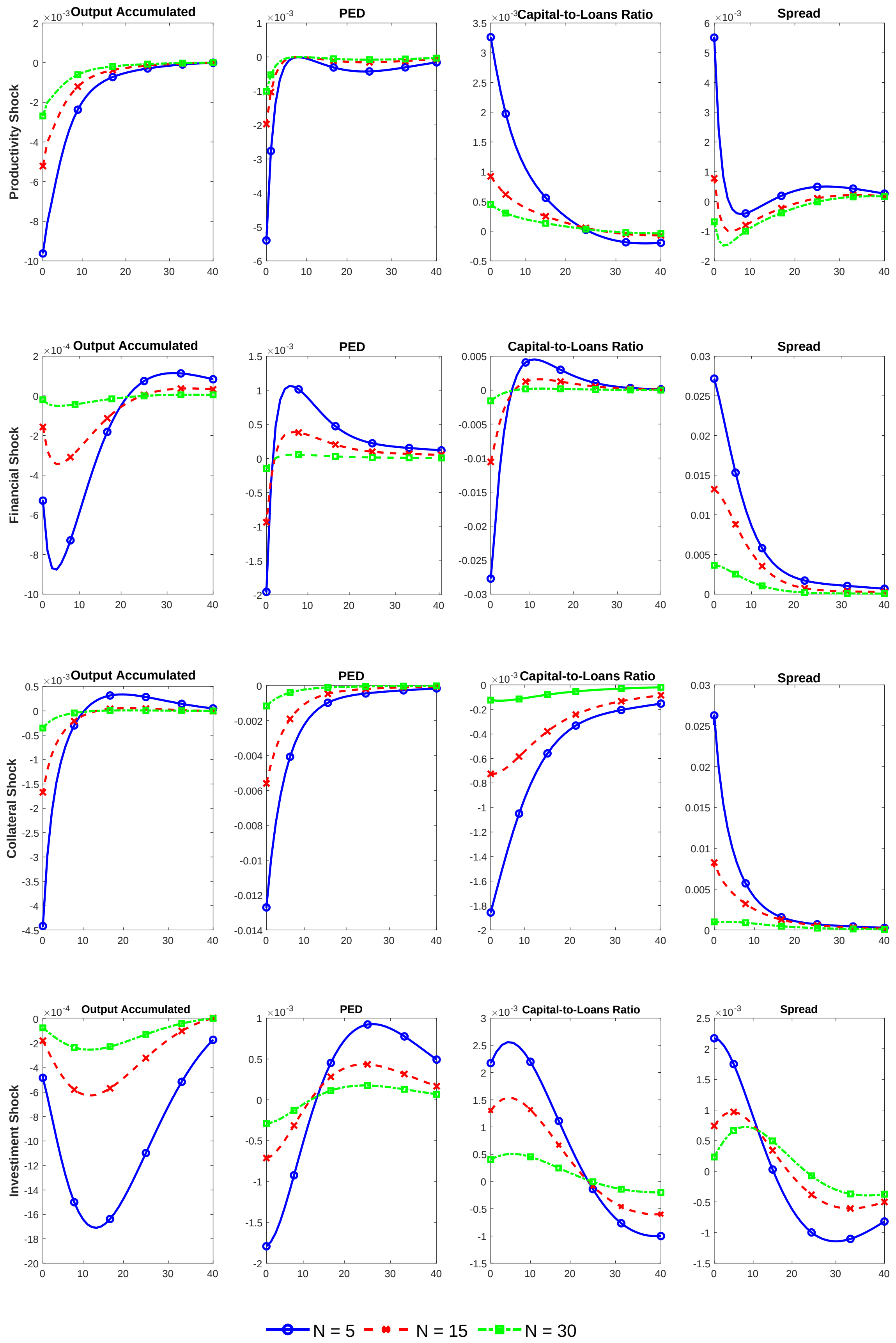
Figure 9 shows the impulse responses of output accumulated y, loan demand elasticity PED, capital-to-loans ratio $\frac{k^B}{b}$ and spread after four types of shocks when the number of banks N is five, fifteen, and thirty (i.e., closer to the perfect competition). When there are only five banks, the amplification effect is much more significant for all variables after all the adverse shocks. For low N, the greater the power of banks' markets to readjust the loan rate R^b after negative shocks and obtain more significant capital accumulation. Consequently, a greater amplifying effect on the spread is transmitted to other variables in the model. The larger spread reduces the activity carried out by entrepreneurs, as the cost of borrowing has become more expensive, causing a sharp drop in the accumulated output of the economy.

If we allow new banks to enter the interbank market until we have the number $N = 15$, the spread amplifying effect decreases considerably. With more banks competing with each other, the value of the loan rate falls, consequently, of the spread. With lower interest on loans, entrepreneurs can leverage more, and PED becomes less inelastic. The increase in PED and borrowing for business activity affects the accumulated output of the economy, attenuating its fall. Suppose we further reduce the barriers to entry for new banks so that bank competition increases and we have $N = 30$. In that case, the spread amplification effect practically disappears, as can be seen in Figure 9.

For the negative financial shock that destroys bank capital k^B, the increase in the number of banks to $N = 30$ causes the spread variation to be close to zero. The reduction in k^B that causes the fall of the capital-to-loans ratio and generates the amplifying effect on the spread is overcome by the impact of perfect banking competition. Banks do not have the market power to adjust the loan rate R^b. In this way, business activity is not harmful. PED is elastic because R^b is low, and consequently, the fall in the economy's accumulated output y is practically inexistent.

Regarding the adverse shocks that pass through the price of capital q, such as the productivity shock and the investment shock, the greater interbank competition reduces the loan rate R^b and, consequently, the spread, makes the PED not fall so significantly. The first effect shifts the elasticity of loans PED down due to the reduction in the price of capital is counterbalanced by the second effect from the lower loan rate, preventing the total fall in the PED from being amplified. Finally, the collateral shock that destroys entrepreneurs' assets causes the elasticity of loans to be reduced due to a smaller amount of capital. However, greater banking competition means that entrepreneurs get more loans at a lower rate, generating a PED increase effect. A more elastic PED prevents entrepreneurs from reducing their activities. Consequently, the fall in accumulated output will be close to zero.

Figure 9: Impulse responses to a negatives shocks with differents N



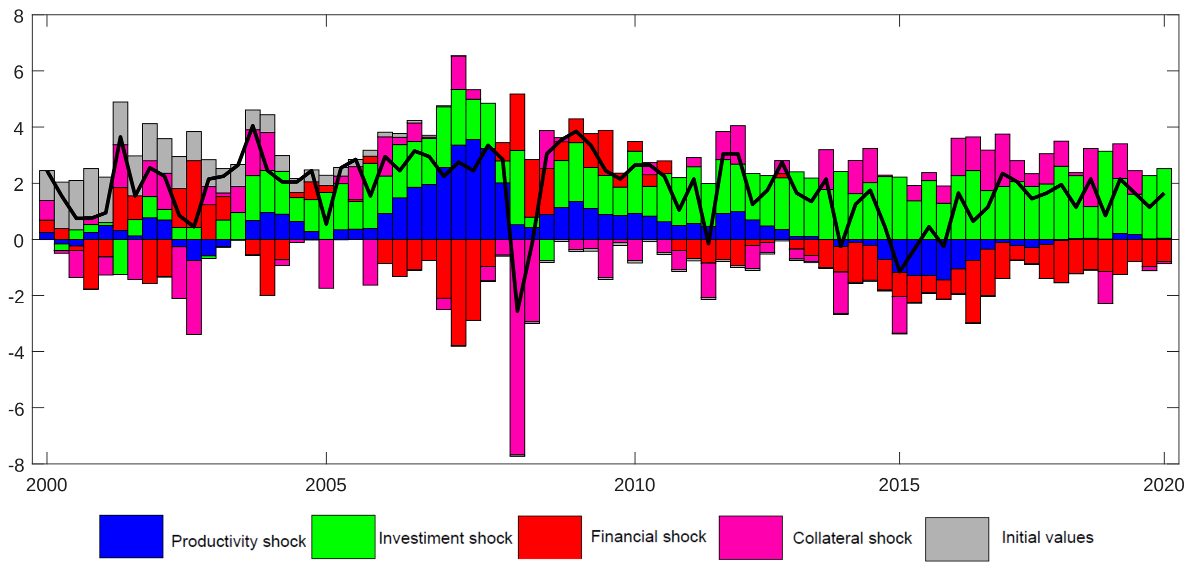
Note: The horizontal axis shows quarters after a negatives shocks at the beginning of period one. The vertical axis shows the percentage deviation from the steady-state for real variables and the spread, which are expressed in deviations from the steady-state in percent points.

8 Historical Decomposition

This section shows how productivity, collateral, financial, and investment shocks explain output and spread fluctuations. Figure 10 shows the decomposition of the accumulated output (deviations from the steady-state) between productivity, investment, financial and collateral shocks. We can see that the shocks mentioned are essential to explain the output dynamics in the selected period. The investment shock acts as a component that drives output increase, especially after the 2008 financial crisis. The productivity shock also played an important role in output increase until 2014. However, with the onset of the Brazilian recession in 2015, the productivity shock contributed to the drop in output.

About the collateral shock that affects the assets of entrepreneurs, it is possible to note that it contributes to increases and a fall in output at different times throughout the selected period. It is possible to state that collateral shock was the main driver for the drop in output during the 2008 financial crisis. In Brazil, countercyclical measures stood out in the fight against the crisis. The domestic market was generally encouraged with increased credit, lower interest rates, and tax cuts. In 2009, it was already possible to notice recovery in the Brazilian economy, both in growth and the return of financial flows to the country. Finally, the financial shock contributes more to the drop in output than to its growth, except during the credit expansion policy that took place in the 2008 crisis. This negative contribution is due to the high concentration in the Brazilian interbank market, which passes on the effects of financial shocks to borrowers through a high loan rate. The period of most significant negative contribution was from 2014 to 2019.

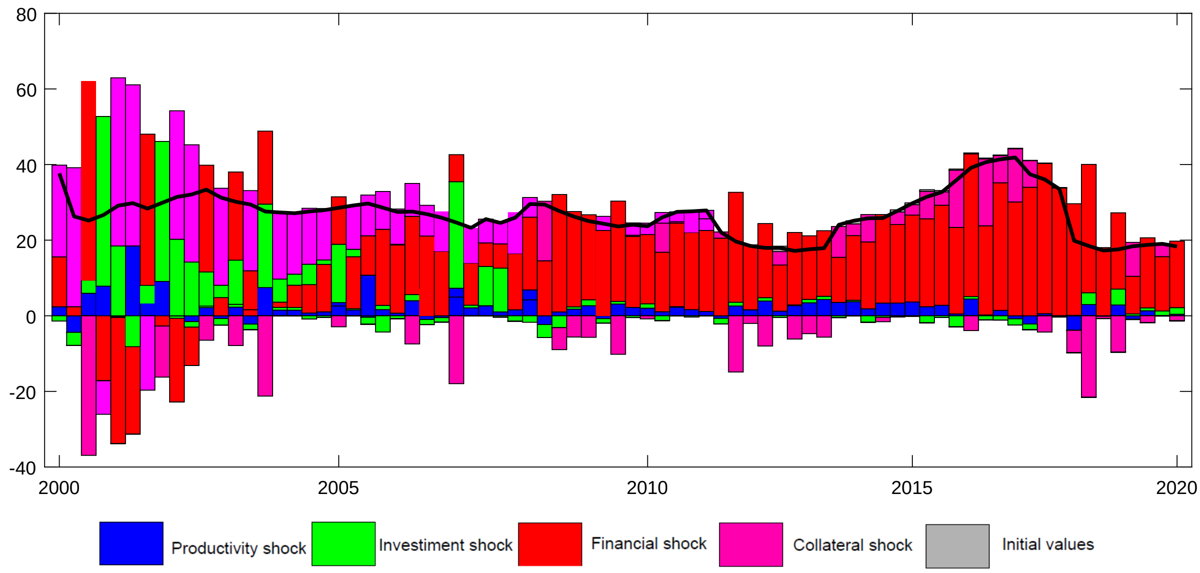
Figure 10: Historical decomposition of the accumulated output (perc. dev. from ss)



Note: The decomposition is computed using the median of the posterior distribution of the benchmark model. Macro shocks include productivity, investment, and collateral shocks. Financial shocks include bank balance sheet shocks.

It is possible to notice the more significant influence of the financial shocks on the spread increases about the spread decomposition in Figure 11. From 2008 onwards, the financial shock becomes dominant for generating spread fluctuations. Financial shocks that hit banks' balance sheets provide the necessary incentive for banks in imperfect competition to readjust the rate charged on loans. Consequently, the increases the spread. Before 2008, the collateral shock also played a more important role in spread fluctuation that declined over time. On the other hand, the productivity shock maintained a practically constant influence on the spread over time. Finally, the investment shock, which had more significant participation in the spread increase during 2000-2008, had its influence reduced after 2008.

Figure 11: Historical decomposition of the spread (perc. dev. from ss)



Note: The decomposition is computed using the median of the posterior distribution of the benchmark model. Macro shocks include productivity, investment, and collateral shocks. Financial shocks include bank balance sheet shocks.

9 Conclusions

This paper studies how imperfect banking competition (Cournot) with bank capital accumulation affects aggregate fluctuations through a time-varying lending rate markup. The model has been estimated using Bayesian techniques and data for Brazil over the period 2000-2019 to analyze important issues of the interbank market. The paper presented a model in which entrepreneurs contract loans subject to borrowing constraints. Loans are provided by imperfectly competitive banks, using deposits collected from savers and bank capital accumulated. Banks' balance sheet constraints establish a link between the business cycle, affecting banks' profits, capital, and the supply and cost of borrowing. We find that in the presence of the cost of capitalization of banks, the amplifying effect of spread tends to be greater than the amplifying effect seen only with imperfect banking competition. This effect tends to amplify the response of output, investment, consumption, and physical capital in the presence of productivity and collateral shocks. At the same time, the amplifying effect is relatively smaller in the presence of investment shocks.

The countercyclical spread, which arises from a joint effect between the elasticity of loans varying over time, the market power of banks, and their cost of capitalization, causes the amplifying effect on the real and financial variables. After adverse shocks that reduce the price of capital, the borrower is more financially constrained, leading to a more inelastic loan elasticity. The collateral shock that reduces entrepreneurs' capital can also reduce the elasticity of loans. Banks with market power can take advantage of this lower loan elasticity by reducing the number of loans offered to entrepreneurs. In this way, banks can find a higher equilibrium loan rate, consequently increasing the spread. About the financial shocks that affect banks' capital, the amplifying effect on the spread and, consequently, on the accumulated output and consumption depends on the banks' capitalization cost. The higher this cost, the more significant the spread increase due to financial shocks that move the banks'capital-to-loans ratio away from the target defined by the monetary authority.

Finally, we show that the reduction of barriers to entry in the interbank market and that the access of new banks allows for a reduction in the spread amplifying effect and reduces the adverse effects of shocks on the real economy. The negative variations in the accumulated output, consumption, and investment are smaller due to more banks in the market. We also show that most of the spread increase in Brazil is due to financial shocks, mainly after 2008. The collateral shocks, which once had a more significant influence on spread fluctuations, reduced their participation after 2008. The financial shocks that increase the spread contribute for the most part to the fall in accumulated output in Brazil. With the onset of the Brazilian recession in 2015, the productivity shock also contributed to the drop in output. Conversely, investment shocks play a different role, mainly contributing to output growth in 2008-2019.

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Appendices

A Households' Optimization Problem

The representative household maximizes their utility subject to the budget constraint (49):

$$\begin{aligned} \max_{\{c_t, l_t, d_t\}} \quad & E_t \sum_{s=0}^{\infty} \beta^s \left[\ln(c_{t+s}) + \phi_l \ln(1 - l_{t+s})\right] \\ \text{s.t.} \quad & c_t + d_t = \frac{R^d_{t-1} d_{t-1}}{\pi_t} + w_t l_t + \Gamma_t^{CP} + \Gamma_t^R + div_t^B \end{aligned} \tag{49}$$

The lagrangian for this problem can be written as:

$$L = E_t \sum_{s=0}^{\infty} \beta^s \left[\ln(c_{t+s}) + \phi_l \ln(1 - l_{t+s})\right] + \lambda_t \left[\frac{R^d_{t-1} d_{t-1}}{\pi_t} + w_t l_t + \Gamma_t^{CP} + \Gamma_t^R + div_t - c_t - d_t\right]$$

and the foc's are:

$$\begin{aligned} [c_t]: \quad & \frac{1}{c_t} - \lambda_t = 0 \\ & \lambda_t = \frac{1}{c_t} \end{aligned} \tag{50}$$

$$\begin{aligned} [l_t]: \quad & \lambda_t w_t - \phi_l \frac{1}{(1 - l_t)} = 0 \\ & \lambda_t w_t = \frac{\phi_l}{(1 - l_t)} \end{aligned} \tag{51}$$

$$\begin{aligned} [d_t]: \quad & -\lambda_t + \beta E_t \left[\lambda_{t+1} \frac{R^d_t}{\pi_{t+1}}\right] = 0 \\ & \lambda_t = \beta E_t \left[\lambda_{t+1} \frac{R^d_t}{\pi_{t+1}}\right] \end{aligned} \tag{52}$$

where we can do the following simplification in the Euler's equation:

$$1 = E_t \left[\Lambda_{t,t+1} \frac{R^d_t}{\pi_{t+1}}\right] \tag{53}$$

using $\Lambda_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$.

B Entrepreneurs' Optimization Problem

The entrepreneur's objective is to maximize the expected lifetime utility:

$$E_t \sum_{s=0}^{\infty} (\beta^E)^s \ln(c_{t+s}^E) \tag{54}$$

subject to a budget constraint:

$$c_t^E + q_t k_t + w_t l_t + \frac{R_{t-1}^b b_{t-1}}{\pi_t} = \frac{y_t^w}{x_t} + (1-\delta) q_t k_{t-1} + b_t \tag{55}$$

and subject to a borrowing constraint:

$$b_t \leq m_t^k E_t \left[\frac{q_{t+1}(1-\delta) k_t \pi_{t+1}}{R_t^b} \right] \tag{56}$$

which $m_t^k \in (0,1)$ denote the fractions of physical capital collateral that can be confiscated by banks when the entrepreneurs fail to repay their debt. The entrepreneurs' lagrangian can be written as:

$$L = E_t \sum_{s=0}^{\infty} \left(\beta^E\right)^s \ln\left(c_{t+s}^E\right) - \lambda_{1,t}^E \left\{ c_t^E + q_t k_t + w_t l_t + \frac{R_{t-1}^b b_{t-1}}{\pi_t} - \frac{y_t^w}{x_t} - (1-\delta) q_t k_{t-1} - b_t \right\}$$
$$- \lambda_{2,t}^E \left\{ b_t - m_t^k E_t \left[\frac{q_{t+1} k_t (1-\delta) \pi_{t+1}}{R_t^b} \right] \right\}$$

The foc's are:

$$(c_t^E): \quad \frac{1}{c_t^E} - \lambda_{1,t}^E = 0$$
$$\lambda_{1,t}^E = \frac{1}{c_t^E} \tag{57}$$

$$(b_t): -\lambda_{2,t}^E - \beta^E E_t \left[\lambda_{1,t+1}^E \frac{R_t^b}{\pi_{t+1}} \right] + \lambda_{1,t}^E = 0$$
$$\lambda_{2,t}^E = \lambda_{1,t}^E - \beta^E E_t \left[\lambda_{1,t+1}^E \frac{R_t^b}{\pi_{t+1}} \right] \tag{58}$$

$$(l_t): -\lambda_{1,t}^E w_t + \lambda_{1,t}^E (1-\alpha) \frac{z_t (k_{t-1})^{\alpha} (l_t)^{-\alpha}}{x_t} \frac{l_t}{l_t} = 0$$
$$w_t = (1-\alpha) \frac{y_t^w}{x_t l_t} \tag{59}$$

$$(k_t): -\lambda^E_{1,t}q_t + \beta^E E_t\left[\lambda^E_{1,t+1}\frac{\alpha z_{t+1}(k_t)^{\alpha-1}(l_{t+1})^{1-\alpha}}{x_{t+1}}\frac{k_t}{k_t}\right] + \beta^E E_t[\lambda^E_{1,t+1}(1-\delta)q_{t+1}]$$
$$+\lambda^E_{2,t}E_t\left[\frac{m^k_t(1-\delta)q_{t+1}\pi_{t+1}}{R^b_t}\right] = 0$$
$$\lambda^E_{1,t}q_t = \beta^E E_t\left[\lambda^E_{1,t+1}\left(\alpha\frac{y^w_{t+1}}{x_{t+1}k_t} + (1-\delta)q_{t+1}\right)\right] + \lambda^E_{2,t}E_t\left[\frac{m^k_t(1-\delta)q_{t+1}\pi_{t+1}}{R^b_t}\right] \quad (60)$$

It is possible to find an expression for k_t replacing (56), (57) and (58) in (60):

$$\frac{q_t}{c^E_t} = \beta^E E_t\left[\frac{1}{c^E_{t+1}}\left(\alpha\frac{y^w_{t+1}}{x_{t+1}k_t} + (1-\delta)q_{t+1}\right)\right] + \left[\frac{1}{c^E_t} - \beta^E E_t\left(\frac{R^b_t}{c^E_{t+1}}\right)\right]E_t\left[\frac{m^k_t(1-\delta)q_{t+1}\pi_{t+1}}{R^b_t}\right]$$
$$q_t = \beta^E E_t\left[\frac{c^E_t}{c^E_{t+1}}\left(\alpha\frac{y^w_{t+1}}{x_{t+1}k_t} + (1-\delta)q_{t+1}\right)\right] + c^E_t\left[\frac{1}{c^E_t} - \beta^E E_t\left(\frac{R^b_t}{c^E_{t+1}}\right)\right]\frac{b_t}{k_t} \quad (61)$$

C Capital Producers' Optimization Problem

Capital producers buy the non-depreciated capital from entrepreneurs and the final good from retailers to produce a new capital sold to entrepreneurs. The capital producers' optimization problem can be written as:

$$\begin{aligned} \max_{\{i_t,k_t\}} \quad & E_t\sum_{s=0}^{\infty}\Lambda_{t,t+s}\left[q_tk_t - q_t(1-\delta)k_{t-1} - i_t - \frac{\chi}{2}\left(\frac{i_ts^{qk}_t}{i_{t-1}} - 1\right)^2 i_t\right] \\ \text{s.t.} \quad & k_t = (1-\delta)k_{t-1} + i_t \end{aligned} \quad (62)$$

where $\Lambda_{t,t+s} \equiv \beta^s\frac{u'(c_{t+s})}{u'(c_t)}$ is the stochastic discount factor since the households are themselves the capital producers. The objective function (62) can be simplified to:

$$\max_{\{i_t,k_t\}} E_t\sum_{s=0}^{\infty}\Lambda_{t,t+s}\left\{q_t[k_t-(1-\delta)k_{t-1}] - i_t - \frac{\chi}{2}\left(\frac{i_ts^{qk}_t}{i_{t-1}} - 1\right)^2 i_t\right\}$$
$$\max_{\{i_t,k_t\}} E_t\sum_{s=0}^{\infty}\Lambda_{t,t+s}\left\{q_ti_t - i_t - \frac{\chi}{2}\left(\frac{i_ts^{qk}_t}{i_{t-1}} - 1\right)^2 i_t\right\}$$
$$\max_{\{i_t\}} E_t\sum_{s=0}^{\infty}\Lambda_{t,t+s}\left\{(q_t-1)i_t - \frac{\chi}{2}\left(\frac{i_ts^{qk}_t}{i_{t-1}} - 1\right)^2 i_t\right\} \quad (63)$$

The first order condition in relation to i_t is:

$$(q_t-1) - \frac{\chi}{2}\left(\frac{i_ts^{qk}_t}{i_{t-1}} - 1\right)^2 - i_t\left\{\chi\left(\frac{i_ts^{qk}_t}{i_{t-1}} - 1\right)\frac{s^{qk}_t}{i_{t-1}}\right\} - E_t\left\{-\Lambda_{t,t+1}i_{t+1}\chi\left(\frac{i_{t+1}s^{qk}_{t+1}}{i_t} - 1\right)\frac{i_{t+1}s^{qk}_{t+1}}{i_t^2}\right\} = 0$$

Thus, the price of capital q_t is:

$$1 = q_t - \frac{\chi}{2}\left(\frac{i_t s_t^{qk}}{i_{t-1}} - 1\right)^2 - i_t\left\{\chi\left(\frac{i_t s_t^{qk}}{i_{t-1}} - 1\right)\frac{s_t^{qk}}{i_{t-1}}\right\} + E_t\left\{\Lambda_{t,t+1} i_{t+1}\chi\left(\frac{i_{t+1}s_{t+1}^{qk}}{i_t} - 1\right)\frac{i_{t+1}s_{t+1}^{qk}}{i_t^2}\right\}$$

$$q_t = 1 + \frac{\chi}{2}\left(\frac{i_t s_t^{qk}}{i_{t-1}} - 1\right)^2 + \chi\left(\frac{i_t s_t^{qk}}{i_{t-1}} - 1\right)\left(\frac{i_t}{i_{t-1}}\right)s_t^{qk} - \chi E_t\left\{\Lambda_{t,t+1}\left(\frac{i_{t+1}s_{t+1}^{qk}}{i_t} - 1\right)\left(\frac{i_{t+1}}{i_t}\right)^2 s_{t+1}^{qk}\right\} \quad (64)$$

And, the profits of capital producers can be written as:

$$\Gamma_t^{CP} = (q_t - 1)i_t - \frac{\chi}{2}\left(\frac{i_t s_t^{qk}}{i_{t-1}} - 1\right)^2 i_t \quad (65)$$

D Retailers' Optimization Problem

We know that the problem of the representative retail firm producing the consumption final good y_t can be written as:

$$\Gamma^R = E_t\sum_{t=0}^{\infty}\Lambda_{t,t+s}\left[\frac{p_t(i)^{1-\epsilon_t^y}y_t}{p_t} - \frac{p_t^w(i)p_t(i)^{-\epsilon_t^y}y_t}{p_t} - \frac{\kappa_\pi}{2}\left(\frac{p_t(i)}{p_{t-1}(i)} - \pi_{t-1}^{\iota_p}\overline{\pi}^{1-\iota_p}\right)^2 p_t y_t\right] \quad (66)$$

The first-order condition in relation to $p_t(i)$ is:

$$\begin{aligned}(1-\epsilon_t^y)\frac{p_t(i)^{-\epsilon_t^y}y_t}{p_t} + \epsilon_t^y\frac{p_t^w(i)p_t(i)^{-\epsilon_t^y-1}y_t}{p_t} - \kappa_\pi\left(\frac{p_t(i)}{p_{t-1}(i)} - \pi_{t-1}^{\iota_p}\overline{\pi}^{1-\iota_p}\right)\frac{p_t y_t}{p_{t-1}(i)}\\ -\beta E_t\left[\Lambda_{t,t+1}\kappa_\pi\left(\frac{p_{t+1}(i)}{p_t(i)} - \pi_t^{\iota_p}\overline{\pi}^{1-\iota_p}\right)p_{t+1}y_{t+1}\left(-\frac{p_{t+1}(i)}{p_t(i)^2}\right)\right] = 0 \quad (67)\end{aligned}$$

Dividing the expression above by y_t:

$$\begin{aligned}(1-\epsilon_t^y)\frac{p_t(i)^{-\epsilon_t^y}}{p_t} + \epsilon_t^y\frac{p_t^w(i)p_t(i)^{-\epsilon_t^y-1}}{p_t} - \kappa_\pi\left(\frac{p_t(i)}{p_{t-1}(i)} - \pi_{t-1}^{\iota_p}\overline{\pi}^{1-\iota_p}\right)\frac{p_t}{p_{t-1}(i)}\\ -\beta E_t\left[\Lambda_{t,t+1}\kappa_\pi\left(\frac{p_{t+1}(i)}{p_t(i)} - \pi_t^{\iota_p}\overline{\pi}^{1-\iota_p}\right)p_{t+1}\frac{y_{t+1}}{y_t}\left(-\frac{p_{t+1}(i)}{p_t(i)^2}\right)\right] = 0 \quad (68)\end{aligned}$$

In symmetrical equilibrium, or $p_t(i) = p_t$, first order conditions imply the Phillips curve nonlinear, given by:

$$(1-\epsilon_t^y) + \frac{\epsilon_t^y}{x_t} - \kappa_\pi\left(\pi_t - \pi_{t-1}^{\iota_p}\overline{\pi}^{1-\iota_p}\right)\pi_t + \beta E_t\left[\Lambda_{t,t+1}\kappa_\pi\left(\pi_{t+1} - \pi_t^{\iota_p}\overline{\pi}^{1-\iota_p}\right)\pi_{t+1}^2\frac{y_{t+1}}{y_t}\right] = 0 \quad (69)$$

where $x_t = \frac{p_t(i)}{p_t^w(i)} = mc_t(i)$ is the mark-up of the final good price.

E Banks' Optimization Problem with Capital Accumulation

Assume that banks are owned by an households who receives real dividends and spends this dividend with consumption: $div_t^B(j) = c_t$. The banker j's utility function is $U_t = \ln c_t$. Thus, the bank j's problem can be written as:

$$\begin{aligned}
\max_{\{b_t(j), k_t^b(j), div_t^B(j)\}} \quad & E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}[\ln(div_{t+s}^B(j))] \\
\text{s.t.} \quad & k_t^B(j) = (1-\delta^B)k_{t-1}^B(j) + \Gamma_t^B(j) - div_t^B(j) \\
& div_t^B(j) \geq 0 \\
& k_t^B(j) \geq 0 \\
& b_t(j) \geq 0
\end{aligned} \tag{70}$$

The lagrangian is:

$$L = \sum_{s=0}^{\infty} \Lambda_{t,t+1} E_t \left\{ \ln(div_t^B(j)) + \lambda_t^B(j) \left[k_t^B(j) + div_t^B(j) - (1-\delta^B)k_{t-1}^B(j) - \Gamma_t^B(j) \right] \right\}$$

Thus, the foc's are:

$$\begin{aligned}
[div_t^B(j)] : \frac{1}{div_t^B(j)} + \lambda_t^B(j) &= 0 \\
\lambda_t^B(j) &= -\frac{1}{div_t^B(j)}
\end{aligned} \tag{71}$$

$$[b_t(j)] : E_t \Lambda_{t,t+1} \left\{ \frac{\lambda_{t+1}^B(j)}{\pi_{t+1}} \left[\frac{\partial \Omega_t^B(j)}{\partial b_t(j)} - \left(\frac{\partial R_t^b}{\partial b_t(j)} b_t(j) + R_t^b - R_t^d \right) \right] \right\} = 0 \tag{72}$$

$$[k_t^B(j)] : \lambda_t^B(j) + E_t \lambda_{t+1}^B(j) \left\{ \frac{1}{\pi_{t+1}} \left[\frac{\partial \Omega_t^B(j)}{\partial k_t^B(j)} - R_t^d \right] - (1-\delta^B) \right\} = 0 \tag{73}$$

Then using $\frac{\partial R_t^b}{\partial b_t(j)} = \frac{\partial R_t^b}{\partial b_t} \frac{\partial b_t}{\partial b_t(j)} = \frac{\partial R_t^b}{\partial b_t}$ in (72), we get the following expression:

$$\begin{aligned}
\frac{\partial \Omega_t^B(j)}{\partial b_t(j)} - \left(\frac{\partial R_t^b}{\partial b_t(j)} b_t(j) + R_t^b - R_t^d \right) &= 0 \\
\frac{\partial \Omega_t^B(j)}{\partial b_t(j)} - \left(\frac{\partial R_t^b}{\partial b_t} \frac{b_t}{N} + R_t^b - R_t^d \right) &= 0 \\
\frac{\partial \Omega_t^B(j)}{\partial b_t(j)} - \left(\frac{\partial R_t^b}{\partial b_t} \frac{b_t}{R_t^b} \frac{1}{N} + 1 \right) R_t^b + R_t^d &= 0 \\
\frac{\partial \Omega_t^B(j)}{\partial b_t(j)} - \left(1 - PED_t^{-1} \frac{1}{N} \right) R_t^b + R_t^d &= 0
\end{aligned}$$

Thus, isolating R_t^b and using $b_t = \frac{b_t(j)}{N}$ and $k_t^B = \frac{k_t^B(j)}{N}$, we get the loan interest rate R_t^b:

$$
\begin{aligned}
R_t^b &= \frac{R_t^d + \dfrac{\partial \Omega_t^B(j)}{\partial b_t(j)}}{\left(1 - PED_t^{-1}\dfrac{1}{N}\right)} \\
R_t^b &= \frac{R_t^d - \kappa_{k^B}\left(\dfrac{k_t^B(j)}{b_t(j)} - \tau^B\right)\left(\dfrac{k_t^B(j)}{b_t(j)}\right)^2}{\left(1 - PED_t^{-1}\dfrac{1}{N}\right)} \\
R_t^b &= \frac{R_t^d - \kappa_{k^B}\left(\dfrac{k_t^B}{b_t} - \tau^B\right)\left(\dfrac{k_t^B}{b_t}\right)^2}{\left(1 - PED_t^{-1}\dfrac{1}{N}\right)} \qquad (74)
\end{aligned}
$$

and, come backing to (73):

$$
\begin{aligned}
\lambda_t^B(j) + E_t\lambda_{t+1}^B(j)\left\{\frac{1}{\pi_{t+1}}\left[\frac{\partial \Omega_t^B(j)}{\partial k_t^B(j)} - R_t^d\right] - (1-\delta^B)\right\} &= 0 \\
E_t\Lambda_{t,t+1}\left\{\frac{1}{div_{t+1}^B(j)}\right\}\left\{(1-\delta^B) + \frac{1}{\pi_{t+1}}\left[R_t^d - \frac{\partial \Omega_t^B(j)}{\partial k_t^B(j)}\right]\right\} &= \frac{1}{div_t^B(j)}
\end{aligned}
$$

then:

$$
E_t\Lambda_{t,t+1}\left\{\frac{1}{div_{t+1}^B(j)}\right\}\left\{(1-\delta^B) + \frac{1}{\pi_{t+1}}\left[R_t^d - \kappa_{k^B}\left(\frac{k_t^B(j)}{b_t(j)} - \tau^B\right)\left(\frac{k_t^B(j)}{b_t(j)}\right) - \frac{\kappa_{k^B}}{2}\left(\frac{k_t^B(j)}{b_t(j)} - \tau^B\right)^2\right]\right\} = \frac{1}{div_t^B(j)}
$$

rearranging the above terms and using again $b_t = \frac{b_t(j)}{N}$ and $k_t^B = \frac{k_t^B(j)}{N}$ with $div_t^B = \frac{div_t^B(j)}{N}$, we get:

$$
\begin{aligned}
E_t\Lambda_{t,t+1}\left\{\frac{1}{div_{t+1}^B}\right\}\left\{(1-\delta^B) + \frac{1}{\pi_{t+1}}\left[R_t^d - \kappa_{k^B}\left(\frac{k_t^B}{b_t} - \tau^B\right)\left(\frac{3}{2}\left(\frac{k_t^B}{b_t}\right) - \frac{\tau^B}{2}\right)\right]\right\} &= \frac{1}{div_t^B} \\
div_t^B = \left(E_t\Lambda_{t,t+1}\left\{\frac{1}{div_{t+1}^B}\right\}\left\{(1-\delta^B) + \frac{1}{\pi_{t+1}}\left[R_t^d - \kappa_{k^B}\left(\frac{k_t^B}{b_t} - \tau^B\right)\left(\frac{3}{2}\left(\frac{k_t^B}{b_t}\right) - \frac{\tau^B}{2}\right)\right]\right\}\right)^{-1} & \qquad (75)
\end{aligned}
$$

where $\Lambda_{t,t+1} = \beta\frac{u'(c_{t+1})}{u'(c_t)}$.

F Calculating the Elasticity of Loan Demand to the Loan Rate (PED)

The PED can be calculated replacing (57) and (58) in (60):

$$
\begin{aligned}
\frac{1}{c_t^E} q_t &= \beta^E E_t \left[\frac{1}{c_{t+1}^E} \left(\alpha \frac{y_{t+1}^w}{x_{t+1} k_t} + (1-\delta) q_{t+1} \right) \right] + \left(\frac{1}{c_t^E} - \beta^E E_t \left[\frac{1}{c_{t+1}^E} \frac{R_t^b}{\pi_{t+1}} \right] \right) m_t^k E_t \left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b} \right] \\
\frac{1}{c_t^E} q_t &= \beta^E E_t \left[\frac{1}{c_{t+1}^E} \left(\alpha \frac{y_{t+1}^w}{x_{t+1} k_t} + (1-\delta) q_{t+1} \right) \right] + \frac{1}{c_t^E} m_t^k E_t \left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b} \right] \\
&- \beta^E E_t \left[\frac{1}{c_{t+1}^E} \frac{R_t^b}{\pi_{t+1}} \right] m_t^k E_t \left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b} \right]
\end{aligned}
$$

Thus,

$$
\frac{1}{c_t^E} \left(q_t - m_t^k E_t \left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b} \right] \right) = \beta^E E_t \left[\frac{1}{c_{t+1}^E} \left(\alpha \frac{y_{t+1}^w}{x_{t+1} k_t} + (1-\delta) q_{t+1} - m_t^k E_t [q_{t+1}(1-\delta)] \right) \right] \tag{76}
$$

Using (7) in (76):

$$
\begin{aligned}
q_t - m_t^k E_t \left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b} \right] &= \beta^E E_t \left[\frac{c_t^E}{c_{t+1}^E} \frac{\alpha z_{t+1} (l_{t+1})^{1-\alpha}}{x_{t+1}} (k_t)^{\alpha-1} \right] \\
&+ \beta^E E_t \left[(1-\delta) q_{t+1} - m_t^k E_t \left[q_{t+1}(1-\delta) \right] \right]
\end{aligned} \tag{77}
$$

Define now:

$$
A_{k,t} \equiv q_t - m_t^k E_t \left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b} \right] \tag{78}
$$

$$
B_{k,t} \equiv \beta^E E_t \left[\frac{c_t^E}{c_{t+1}^E} \frac{\alpha z_{t+1} (l_{t+1})^{1-\alpha}}{x_{t+1}} \right] > 0 \tag{79}
$$

$$
C_{k,t} \equiv \beta^E E_t \left[(1-\delta) q_{t+1} - m_t^k E_t \left[q_{t+1}(1-\delta) \right] \right] \tag{80}
$$

Then, we can write (77) as:

$$
A_{k,t} = B_{k,t} (k_t)^{\alpha-1} + C_{k,t} \tag{81}
$$

Isolating capital in the above equation:

$$
\frac{A_{k,t} - C_{k,t}}{B_{k,t}} = (k_t)^{\alpha-1}
$$

$$
\left(\frac{A_{k,t} - C_{k,t}}{B_{k,t}} \right)^{\frac{1}{\alpha-1}} = k_t \tag{82}
$$

We know that R_t^b is present in $A_{k,t}$, then deriving (82) in relation to R_t^b:

$$
\frac{\partial k_t}{\partial R_t^b} = \frac{1}{\alpha - 1} \left(\frac{A_{k,t} - C_{k,t}}{B_{k,t}} \right)^{\frac{1}{\alpha-1} - 1} \left(\frac{\partial A_{t,k}}{\partial R_t^b} \frac{1}{B_{k,t}} \right) \tag{83}
$$

The derivative $\frac{\partial A_{t,k}}{\partial R_t^b}$ is equal to:

$$\frac{\partial A_{t,k}}{\partial R_t^b} = m_t^k E_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{(R_t^b)^2}\right] \equiv D_{k,t} > 0 \tag{84}$$

Thus, returning in (83) with $u_1 = \frac{1}{\alpha-1}$:

$$\begin{aligned}
\frac{\partial k_t}{\partial R_t^b} &= u_1\left(\frac{A_{k,t}-C_{k,t}}{B_{k,t}}\right)^{u_1-1}\left(\frac{D_{k,t}}{B_{k,t}}\right) \\
&= u_1\left(\frac{A_{k,t}-C_{k,t}}{B_{k,t}}\right)^{u_1}\left(\frac{A_{k,t}-C_{k,t}}{B_{k,t}}\right)^{-1}\left(\frac{D_{k,t}}{B_{k,t}}\right) \\
&= u_1\left(\frac{A_{k,t}-C_{k,t}}{B_{k,t}}\right)^{u_1}\left(\frac{D_{k,t}}{A_{k,t}-C_{k,t}}\right) \\
&= u_1\left(\frac{D_{k,t}}{A_{k,t}-C_{k,t}}\right) k_t < 0 \qquad (85)
\end{aligned}$$

We will have the negative derivative since $A_{k,t} - C_{k,t} > 0$ (it has already been shown that $u_1 < 0$ and $D_{k,t} > 0$). This condition can be guaranteed by (81):

$$A_{k,t} - C_{k,t} = B_{k,t}(k_t)^{\alpha-1} > 0 \tag{86}$$

Then, taking the derivate of b_t in binding borrowing constraint with respect to R_t^b, we get:

$$\frac{\partial b_t}{\partial R_t^b} = m_t^k E_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_{b,t}}\right]\frac{\partial k_t}{\partial R_t^b} < 0 \tag{87}$$

The market loan demand b_t is downward-slopping in R_t^b. To find an expression for PED_t, elasticity of loan market b_t to the loan rate R_t^b, we do:

$$PED_t \equiv -\frac{\partial b_t}{\partial R_t^b}\frac{R_t^b}{b_t} \equiv -\frac{R_t^b}{b_t}\frac{\partial b_t}{\partial R_t^b} \tag{88}$$

We know that:

$$b_t = m_t^k E_t\left[\frac{q_{t+1}(1-\delta)k_t\pi_{t+1}}{R_t^b}\right] = \left(\frac{1}{R_t^b}\right) m_t^k E_t[q_{t+1}(1-\delta)k_t\pi_{t+1}]$$

Thus,

$$\begin{aligned}
\frac{\partial b_t}{\partial R_t^b} &= -\left(\frac{1}{(R_t^b)^2}\right) m_t^k E_t[q_{t+1}(1-\delta)k_t\pi_{t+1}] + \left(\frac{1}{R_t^b}\right) m_t^k E_t[q_{t+1}(1-\delta)\pi_{t+1}]\left(\frac{\partial k_t}{\partial R_t^b}\right) \\
&= -\left(\frac{1}{R_t^b}\right) m_t^k E_t\left[\frac{q_{t+1}(1-\delta)k_t\pi_{t+1}}{R_t^b}\right] + m_t^k E_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b}\right]\left(\frac{\partial k_t}{\partial R_t^b}\right) \\
&= -\frac{b_t}{R_{b,t}} + m_t^k E_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b}\right]\left(\frac{\partial k_t}{\partial R_t^b}\right) \qquad (89)
\end{aligned}$$

Replacing in (88):

$$\begin{aligned} PED_t &= -\frac{R_t^b}{b_t}\left[-\frac{b_t}{R_t^b} + m_t^k E_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b}\right]\left(\frac{\partial k_t}{\partial R_t^b}\right)\right] \\ &= 1 - \frac{R_t^b}{b_t}\left[m_t^k E_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b}\right]\left(\frac{\partial k_t}{\partial R_t^b}\right)\right] \\ &= 1 - \frac{R_t^b}{b_t}\left[m_t^k E_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b}\right]\frac{k_t}{k_t}\left(\frac{\partial k_t}{\partial R_t^b}\right)\right] \\ &= 1 - \frac{R_t^b}{b_t}\frac{b_t}{k_t}\left(\frac{\partial k_t}{\partial R_t^b}\right) \\ &= 1 + PEK_t > 0 \end{aligned} \tag{90}$$

where $PEK_t \equiv -\frac{\partial k_t}{\partial R_t^b}\frac{R_t^b}{k_t}$ denote the elasticity of entrepreneurs' capital demand to the loan rate. The $PED_t \equiv -\frac{R_t^b}{b_t}\frac{\partial b_t}{\partial R_t^b} > 0$ because $\frac{\partial b_t}{\partial R_t^b} < 0$ and the entrepreneurs' demand for capital decreases with increases in loan rate, $\frac{\partial k_t}{\partial R_t^b} < 0$. Before define:

$$MPK_t \equiv \frac{\alpha z_t (k_{t-1})^{\alpha-1}(l_t)^{1-\alpha}}{x_t} \tag{91}$$

as the marginal product of capital in terms of the final good. And:

$$\Lambda_{t,t+1}^E \equiv \beta^E \frac{u'(c_{t+1}^E)}{u'(c_t^E)} = \beta^E \frac{c_t^E}{c_{t+1}^E} \tag{92}$$

as a stochastic discount factor for entrepreneurs. Thus,

$$PEK_t \equiv -\frac{\partial k_t}{\partial R_t^b}\frac{R_t^b}{k_t} = u_1\left(\frac{D_{k,t}}{A_{k,t} - C_{k,t}}\right)k_t\frac{R_t^b}{k_t} = u_1\left(\frac{D_{k,t}}{A_{k,t} - C_{k,t}}\right)R_t^b \tag{93}$$

which $u_1 = \frac{1}{\alpha - 1} < 0$. Thus, we can written PEK as:

$$PEK_t = -u_1\left(\frac{D_{k,t}}{B_{k,t}(k_t)^{\alpha-1}}\right)R_t^b = -u_1\left(\frac{m_t^k E_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{(R_t^b)^2}\right]}{\beta^E E_t\left[\frac{c_t^E}{c_{t+1}^E}\frac{\alpha z_{t+1}(l_{t+1})^{1-\alpha}}{x_{t+1}}\right](k_t)^{\alpha-1}}\right)R_t^b$$

And,

$$PEK - u_1\left(\frac{m_t^k E_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{(R_t^b)^2}\right]}{E_t\left[\Lambda_{t,t+1}^E\frac{\alpha z_{t+1}(l_{t+1})^{1-\alpha}}{x_{t+1}}\right](k_t)^{\alpha-1}}\right)R_t^b = -u_1\left(\frac{m_t^k E_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b}\right]}{E_t\left[\Lambda_{t,t+1}^E MPK_{t+1}\right]}\right)$$

Thus,

$$PEK = \frac{1}{1-\alpha}\left(\frac{m_t^k E_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b}\right]}{E_t\left[\Lambda_{t,t+1}^E MPK_{t+1}\right]}\right) > 0 \quad (94)$$

It can be seen that PEK_t depend positively on the m_t^k and the expected discounted values of the future prices of capital, $E_t\left[\frac{q_{t+1}(1-\delta)}{R_t^b}\right]$. Furthermore, PEK_t depends negatively on the expected discounted values of the marginal product of capital, $E_t\left[\Lambda_{t,t+1}^E MPK_{t+1}\right]$, in terms of the final good. Replacing (94) in (90), we get that:

$$PED_t = 1 + \frac{1}{1-\alpha}\left(\frac{m_t^k E_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b}\right]}{E_t\left[\Lambda_{t,t+1}^E MPK_{t+1}\right]}\right) > 0 \quad (95)$$

It can be seen that higher values of $E_t\left[\Lambda_{t,t+1}^E MPK_{t+1}\right]$ and also lower values of the $E_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b}\right]$ reduce the elasticity of the loan demand PED_t. In addition, an decrease in m_t^k (after a negative collateral shock) directly reduces PED_t and indirectly by increasing in expected MPK_{t+1} (with the reduction of m_t^k, entrepreneurs obtain less loans and decrease their production, which increases MPK_{t+1}).

G Data and Sources

We use 6 quarterly macroeconomic variables of the Brazilian economy. Data comprises the period between 2000-Q3 to 2019-Q4. Below, we present the chosen variables with their respective sources:

1. Gross domestic product (GDP) - quarter versus immediately previous quarter (%) seasonally adjusted. Source: SCNT from the IBGE;

2. Gross Fixed Capital Formation - quarter versus immediately previous quarter (%) seasonally adjusted. Source: SCNT from the IBGE;

3. Consumer Price Index (IPCA) as a proxy of price inflation. Source: National System of Consumer Price Index (SNIPC) of the IBGE;

4. Interest rate policy (Selic) quarterly. Source: BCB;

5. Loans to entrepreneurs: Credit operations with non-earmarked funds - Consolidate balance (end of period) - Working capital - quarter versus immediately previous quarter (%). Source: BCB;

6. Deposits: Extended payment methods - Deposit money banks - Time deposits, savings and others - quarter versus immediately previous quarter (%). Source:

BCB.

H Prior and Posterior Distributions

Figure 12: Prior and posterior distributions