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Fiscal Risk and Financial Fragility

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Abstract

The crisis experienced by Brazilian states in the recent years highlights the fragility of public finances and the increase in the fiscal risk in Brazil. To control the unsustainable increase of indebtedness, the Fiscal Responsibility Law came into force and established financial restrictions on budget execution with which states had to comply. We propose a new statistical indicator to estimate the probability of noncompliance of the Brazilian states to these financial restrictions that relies on the contingent assets approach originally employed to evaluate firms' probability of default. Our measure innovates in the sense that it incorporates not only historical data like traditional approaches but also a prospective component. We also investigate the resilience of the Brazilian financial sector to the rising fiscal risk of states. Our methodology considers potential contagion in the Brazilian interbank market and credit restrictions to the real sector that can take place in view of bank credit default by states. We find that the impact on the financial system is small, which is in part due to the high levels of provisioning and capitalization maintained by Brazilian banks. Finally, we combine the probability of noncompliance and the corresponding impact on the financial sector to compose a state-specific statistical indicator of expected loss in the financial sector.

Keywords: fiscal risk, public indebtedness, banking, Fiscal Responsibility Law, contagion, probability of noncompliance.
JEL classification: G21, G28, C63.

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1 Introduction

The recent crisis in the state of Rio de Janeiro, also experienced by other neighboring and economically dependent states and even by the federal government, highlights the fragility of public finances at both the state and federal levels in Brazil. The increase in state indebtedness in a context of economic recession that Brazil is facing—with a concomitant sharp fall in revenues—has led the already subdued public finances of Brazilian states to excessively high and worrying levels. Some Brazilian municipalities are also undergoing the same dilemma.

States and municipalities provide a number of public services to society and require resources to undertake investments. These federated entities seek funding to ensure proper functioning of these services to the population. When seeking funding, they often turn to the federated government for guarantees to achieve their objectives. The federated government guarantees allow federated entities to finance themselves at lower interest rates, thus reducing funding costs. Nevertheless, there are rules that impose legal limits to the state indebtedness, which are established by the Fiscal Responsibility Law (FRL), Complementary Law 101/2000.

By achieving high levels of indebtedness or by making staff expenditure excessive in relation to expected revenues, federated entities lose investment capacity, the possibility of receiving voluntary transfers from the federated government and also preclude access to the credit market.

Credit is the driving force of the Brazilian economy. To better understand how fiscal risk in the public sector can impair economic growth, it becomes fundamental to investigate to what extent adverse macroeconomic shocks can disrupt state activities and affect public debt. Despite the importance for public provisioning and government decision-making, there are no theoretical models that allow the calculation of these fiscal risks under a legal framework that binds state debt growth to some financial restrictions.

This paper seeks to fill this theoretical–empirical gap and contributes with the literature on public finance and financial economics in three important points. The first contribution is in the development of a new methodology to estimate the likelihood of noncompliance of states under the legal restrictions imposed by the Fiscal Responsibility Law. The great innovation of this methodology is that it uses not only historical data but also incorporates expectations on macroeconomic conditions. The second is in quantifying the impact that the states would bring on the Brazilian financial system using a systemic risk model that encompasses banks and firms. Third, we then combine the probability of noncompliance and the corresponding impact of the Brazilian states on the financial system to compose a state-specific statistical indicator of expected loss in the financial sector.

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The Brazilian National Treasury estimates the fiscal health of Brazilian states through a rating system that looks at several financial and economic indicators.[1] The method employs collected data from the current and also the previous two years and combine them using a decay weighting scheme. In this way, it performs a backward-looking analysis to assign the fiscal health rating, which we can conceptualize as proportional to the probability of insolvency of that state.

Our methodology estimates the fiscal health of states in a more comprehensive way. We estimate how likely a state is to violate at least one of the financial restrictions that are imposed by the Fiscal Responsibility Law. Such violation has legal consequences both to federated states and to responsible managers. In this way, they are carefully watched by states and are worth to be modeled. We model the probability of noncompliance using the contingent assets model and option theory (Merton (1974)). Our probability of noncompliance has a prospective component in that it is conditional on changes in the real GDP in the near future.[2] As a practical example, our method is able to evaluate how a hypothetical deepening of the Brazilian recession would affect fiscal risk of federated states.

The dependence of the probability of noncompliance on the future real GDP permits us to construct stress scenarios to evaluate the resilience of the states' financial health. If a given scenario is likely to occur, it is possible to take measures to reduce the potential impact—even before it occurs—from a prudential perspective. In this manner, the model allows us to think about fiscal risk in terms of the legal compliance and the balance-sheet deterioration of federated states and thus provides prospective insights for conducting public policies.

We also estimate the resilience of the financial system to credit shocks from the federated states. Due to the very rich data sets maintained by the Central Bank of Brazil, we are able to map out every credit operation of the entire financial and real sectors. The model takes into account the network of loans between financial institutions and federated entities, the interbank market, and also credit operations that banks provide to firms. With this loan-level data, we are able to evaluate the systemic importance of each federated entity to the financial system by verifying the amount of losses that it would impose to the economy in case the federated entity defaults on its outstanding bank credit.

It is important to analyze how resilient the financial system is to credit shocks from the public sector, since it has a central role within an economy. Due to its strong interconnectedness to different market segments, spillover effects can go beyond the financial

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[1]The methodology looks at eight financial and economic indicators: indebtedness, debt service in net current income, primary debt service result, personnel expenses and social charges in net current income, generation of own savings, participation of investments in total expenses, participation of Welfare Policy contributions and remunerations in social security expenditures, tax revenues in costing expenses.

[2]We model the change in states' expected revenue as a function of real GDP, after controlling for other macro variables, such as the exchange rate, interest rate (Selic) and inflation.

system. Thus, a credit default from the public sector can trigger contagion chains inside the interbank market and cause spillovers to the real sector in the form of credit crunches. For instance, the subprime crisis in 2008, which originally started inside the financial system, led to a massive destruction of bank assets, but later also heavily affected external markets, with relevant real effects. Models that analyze possible paths through which shocks can propagate and amplify—either inside and outside the financial system—are welcomed to both the banking literature and regulators, since they have the potential to improve the discussion on how to formulate efficient public policies that aim at designing more resilient systems.

The model also allows us to think about the phenomenon of financial stability in a broader way by also bringing in the role of the government besides banks and firms. Shocks in state finances can negatively reflect on the financial system which, in turn, can amplify these shocks through credit crunches to the real sector and repricing mechanisms. Less credit to the real sector induces lower investment, which can impact firm revenues and thus affect the amount of taxes that the government collects. It is thus important to also analyze the fiscal side of federated entities when dealing with financial stability.

In view of the rapid increase of state indebtedness, it is important to assess to what extent the Brazilian financial system would be resilient to this scenario of rising fiscal risk. Our model sheds light in this issue by not only estimating the level of resilience of the financial system to credit defaults from the public sector but also highlighting which banks would be most affected.

The state-of-the-art discussion of financial stability seeks to develop methodologies to calculate the systemic risk within the financial system, but does not analyze possible complementarities between the financial system and the real side of the economy, at least not at the microeconomic level. Crises arise from the decisions of certain agents who, because of their specificities and interconnectivity, cause other agents to incur stress and eventual losses. These losses lead to a decline in the confidence of economic agents that in turn can potentially develop into systemic negative events.

The link between fiscal risk and financial stability in Brazil is especially relevant due to the high levels of indebtedness of the Brazilian federated entities. Figure 1a portrays the evolution of the consolidated debt of Brazilian federated states broken down by creditor type: the federal government, banks, or other counterparties.

The consolidated debt of Brazilian federated states decreased until 2014, point after which started to rise again. This reversal of the debt trajectory is due in large part to the increase in the indebtedness of all states with the banking system, as Figure 1b reveals. Bank indebtedness coupled with the effects of the recessionary period of the Brazilian economy on revenues aggravates the fiscal situation of the states, leading many of them to exceed the maximum limits of indebtedness imposed by the Fiscal Responsibility Law.

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(a) Consolidated debt	*(b) Outstanding bank debt of federated states*
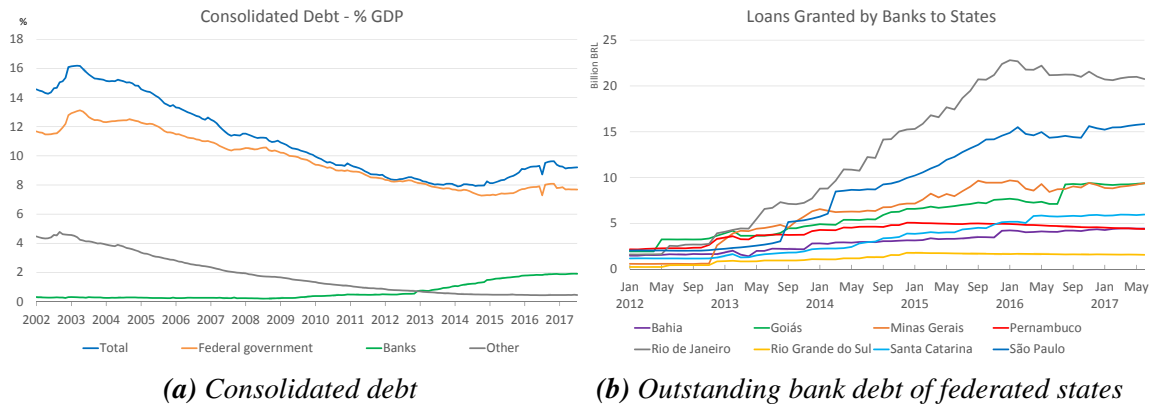
Figure 1: *(a) Consolidated debt of federated states as a percentage of the Brazilian GDP from 2002 to 2017. Total debt of federated states is the sum of outstanding credit from the financial system (bank debt), debt to the federal government (refinanced debt), and other debts. (b) Outstanding bank debt of the most indebtedness states from 2012 to 2017. The initial cutoff point represents the beginning of rising bank debt of Brazilian federated states.*

Given this momentum of rising bank debt by federated states and the negative macroeconomic conditions in Brazil, it becomes important to understand to what extent financial stability could be compromised.

Related literature: Financial stability is a public good that must be safeguarded not only by the national central banks but also by the government as an entirety at a strategic level. Since the 2008 crisis, it has become clear to economists that it is critical to create models that enable them to understand the risks that the financial system incurs and how shocks can unfold over the real economy (Gai et al. (2011)). Models have been developed that help to calculate the systemic risk, which we define as the event in which important institutions enter into a stress regime and lead to a crisis of confidence in the economy. These moments of acute information asymmetry are associated with declines in activity levels, falling asset prices, and abrupt declines in credit.

The study of financial stability requires an understanding of the interconnections that exist in different markets and of how shocks propagate between them (IMF (2015); IMF et al. (2009)). For this reason, studies of complex networks analyzing the interconnections of the financial system and between the latter and the real sector have proliferated (Anand et al. (2015); Aoyama (2014); Bardoscia et al. (2015); Battiston et al. (2016); Blume et al. (2013); Hojman and Szeidl (2008); Lux (2015); Poledna et al. (2015); Silva et al. (2017a, 2016, 2017b); Souza et al. (2016)).

Using complex networks it is possible to analyze how the contagion between institutions inside a network of mutual exposures occurs and what the potential risks and losses that the system as a whole faces (Glasserman and Young (2015)). Still, it also allows to design counterfactual scenarios that evaluate how exogenous shocks—that at first sight can be seen as small—can be amplified and the crisis becomes systemic. To date,

there are no models that analyze the risks of federated entities using complex network theory. This is one of the contributions of this work since it allows an analysis of how these interconnections take place and their effects on the economic system.

It is pertinent to remember that when the Federal Reserve let the Lehman Brothers fail, such firm was considered a small agent with no systemic implications. The crisis that followed the collapse of Lehman Brothers is a clear example of the importance of analyzing the interconnections that exist between financial institutions and the agents that have financial operations with them.

Another relevant case is insurance agencies that may be adversely affected by systemic shocks in the financial system and can amplify shocks so that the entire system collapses. The agents use insurance just to protect themselves from crisis events, but if the insurers themselves can be adversely affected there would be potential for a major crisis. In the case of federated entities, if insurers sell protection against default risk of these entities, shocks or fiscal risks that become systemic can affect that market and adversely affect the financial system with the potential to generate a crisis of great proportions.

2   Indebtedness of Brazilian federated states

State public debt is a recurrent issue in Brazil. In this section, we provide an overview of how state indebtedness evolved in the last decades and the measures that the federal government took in an attempt to control it and ensure public debt sustainability.

Until the mid-1990s, Brazil experienced a persistent inflationary process that culminated in hyperinflation. During this period, public debt control was not tight and states made use of inflationary revenue to obtain primary surpluses. After the implementation of the Plano Real in 1994, inflation stabilized and fiscal problems of the states became more evident.

The states' debt rose sharply due to the reduction of inflationary revenue and to the high interest rates. Consequently, federated states with high indebtedness had problems to roll over their debt in the financial market. To avoid states' defaults, the federated government stepped in to restore their fiscal health. On the one hand, the federal government would assume the states' debt towards the financial market and therefore would became their new creditor. In this debt transformation agreement, the federal government extended the debt maturity up to 30 years and established a lower interest rate. On the other hand, the federated states would commit to restructure their finances, such as in obtaining primary surpluses and privatizing state-owned firms, including banks.

Although the debt restructuring was optional, only two federated states declined this possibility: Amapá and Tocantins. Among the federated states that adhered to this debt restructuring agreement, we highlight São Paulo, Rio de Janeiro, Minas Gerais and Rio

Grande do Sul. They were the most indebted states at that time and are the ones that are struggling the most in the recent debt crisis of Brazilian states.

The high indebtedness of federated states has roots in the structural economic inefficiencies regarding debt sustainability. After the debt restructuring of Brazilian states, the federal government took measures to enforce structural changes to ensure public debt control. To this end, the Fiscal Responsibility Law came into force in 2000 and imposed to public entities new protocols and financial rules.

The Fiscal Responsibility Law established guidelines for budget planning, required the fulfillment of fiscal targets, and enhanced transparency of the entire budget execution process. Not only public entities that are not compliant with the Fiscal Responsibility Law are subject to penalties but also the responsible managers, who can face fines, impeachment and even imprisonment. This accountability of public finances had profound impact on public budget execution.

Among the forbidden actions that would cause state indebtedness, the Fiscal Responsibility Law prohibited credit operations of public entities with controlled companies and restricted the appointment of new expenses to the existence of available resources to fund them.

In addition to the guidelines and actions aimed at improving new public management practices, the Fiscal Responsibility Law imposed a series of limits on personnel expenses[3] and on the levels of indebtedness of the federal government, states and municipalities. In the case of states, the most important limits are:

- *level of indebtedness*: the states' consolidated debt cannot exceed 200 % of their net current revenue.

- *personnel expenses*: states cannot spend more than 60% of the net current revenue with personnel expenses, 49 % for the Executive branch, 3 % for the Legislative branch, 6 % for the Judiciary and 2 % for the public prosecution services.

If states violate one of these legal limits, they must take steps to recomply or otherwise will suffer sanctions, such as the impossibility of receiving further voluntary transfers from the federal government and of engaging in new bank credit operations.

These limits obliged states to take actions to clean up their public accounts so as to recomply with the legal limits. The Fiscal Responsibility Law designed these legal financial limits to contain the unbridled increase in deficits of state accounts and to impose certain fiscal responsibility for all the Brazilian public entities.

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[3]The reduction of personnel costs by public entities faces a number of obstacles, since they cannot reduce salaries and civil servants can only be fired or dismissed in exceptional cases that are listed in the law.

The financial restructuring and the constraints imposed by the Fiscal Responsibility Law combined with a period of economic expansion formed a benign scenario for lowering state debt. The states returned to primary surpluses, thus reducing their debt to GDP. Despite this decrease in public debt, the deterioration in the macroeconomic scenario soon after made the tensions between the federal government and the states awake again.

When Brazil entered a deep recession in 2014, which led to a strong reduction in the state revenues, the fiscal situation of higher indebted states became critical. Some states even delayed salaries to their employees, reduced quality of public services, and substantially reduced their investments.

However, states are not allowed to freely maneuver their budget. For instance, social and political costs allied with the rigidity of the legislation make difficult for states to reduce expenses of federated states. In addition, the Fiscal Responsibility Law prohibited increasing their indebtedness.

In view of this negative macroeconomic scenario, the federal government changed the conditions of the financial restructure to be more beneficial to federated states and to rebalance the growing indebtedness. The federal government extended even more the debt maturity and temporarily suspended in a regressive manner monthly installments to facilitate the recovery of the federated states for 18 months. For the states with the most severe fiscal imbalances, the federal government also proposed a fiscal recovery regime, whose adhesion was not compulsory. In this regime, it further extended the repayment suspension period and let federated entities contract certain credit operations under adverse conditions. States that adhered to the fiscal recovery regime had to implement measures to reduce personnel expenses and tax benefits, to engage in the privatization of public entities, and other measures to guarantee the economic and financial sustainability of public accounts. Rio de Janeiro was the first state to join this regime in September/2017.

3 Methodology

In this section, we discuss the intuition and the underpinnings of our methodology.

3.1 Probability of noncompliance of federated states

We rely on the contingent assets approach, originally employed to model credit risk, to estimate the probability of federal entities to noncomplying to the legal limits of the Fiscal Responsibility Law. We use an adaptation of the Merton (1974)'s classical model that essentially looks at how the components of the balance sheet of firms (states in our case) evolve. The methodology assumes that asset prices evolve stochastically and that the noncompliance occurs when the legal limit that the FRL establishes is violated.

The intuition behind the Merton (1974)'s model is to consider the total assets of a company as the underlying asset of a European purchase option maturing T periods ahead, with an exercise price equal to its obligations. If the company goes bankrupt, shareholders do not get anything. Otherwise, shareholders receive the difference between the values of the assets and the obligations of the companies. Usually, a company goes into bankruptcy before the value of its assets falls below the value of its obligations due to problems of lack of liquidity, for example. Thus, the threshold for estimating a firm's probability of default is less than the value of its debts. For example, the KMV (2001) model sets this threshold limit as the short-term debt plus 50 % of the company's long-term debt.

To adapt this methodology to the case of public sector indebtedness, we consider that there is uncertainty about the evolution of the net current revenue (NCR) over time, rather than the classical approach of using total assets. We attribute changes of the net current revenue to a certainty component (drift) and to an uncertainty component (random or stochastic term). Drift represents the average expected value of revenue growth. The stochastic term is a random walk in which the variance is proportional to the time. Thus, the evolution dynamics of the net current revenue follows a diffusion process:

$$\frac{dNCR}{NCR} = \mu_{NCR}dt + \sigma_{NCR}dZ \tag{1}$$

in which μ_{NCR} is the NCR growth rate (drift), σ_{NCR} is the NCR volatility, and dZ is a Wiener process normally distributed with zero mean and unit variance. For this process, we evaluate the NCR value in $t+1$ as follows:

$$NCR_t = NCR_0 \exp\left[\left(\mu_{NCR} - \frac{\sigma_{NCR}^2}{2}\right)t + \sigma_{NCR}\varepsilon\sqrt{t}\right] \tag{2}$$

in which ε is the realization of a normal random variable with zero mean and unit variance.

Considering that the Fiscal Responsibility Law establishes that the consolidated debt (CD) of the federated entities can not exceed 200 % of the NCR, we estimate the probability that this limit is exceeded in t periods ahead $P(CD_t > 2NCR_t)$ using (2) as follows:

$$P(CD_t > 2NCR_t) = P\left(CD_t > 2NCR_0 \exp\left[\left(\mu_{NCR} - \frac{\sigma_{NCR}^2}{2}\right)t + \sigma_{NCR}\varepsilon\sqrt{t}\right]\right)$$

$$= P\left(\log\left(\frac{CD_t}{2NCR_0}\right) > \left[\left(\mu_{NCR} - \frac{\sigma_{NCR}^2}{2}\right)t + \sigma_{NCR}\varepsilon\sqrt{t}\right]\right)$$

$$= P\left(\log\left(\frac{CD_t}{2NCR_0}\right) - \left(\mu_{NCR} - \frac{\sigma_{NCR}^2}{2}\right)t > \sigma_{NCR}\varepsilon\sqrt{t}\right)$$

$$= P\left(-\frac{\log\left(\frac{2NCR_0}{CD_t}\right) + \left(\mu_{NCR} - \frac{\sigma_{NCR}^2}{2}\right)t}{\sigma_{NCR}\sqrt{t}} > \varepsilon\right) \tag{3}$$

If $d = \dfrac{\log\left(\frac{2NCR_0}{CD_t}\right) + \left(\mu_{NCR} - \frac{\sigma_{NCR}^2}{2}\right)t}{\sigma_{NCR}\sqrt{t}}$, and recalling that $\varepsilon \sim \mathcal{N}(0,1)$, the probability of noncompliance for the legal limit fixed by Fiscal Responsibility Law of the consolidated debt against the net current revenue is:

$$PN_{\text{Debt}} = P\left(\frac{CD_t}{NCR_t} > 200\%\right)$$

$$= P(CD_t > 2 \cdot NCR_t)$$

$$= P(-d > \varepsilon)$$

$$= \mathcal{N}(-d) \tag{4}$$

in which PN_{Debt} is the probability of noncompliance of the consolidated debt with regard to the net current revenue t periods ahead of time.

The value $-d$ measures, in standard deviations of the net current revenue, whether the state will be outside the limits established by the Fiscal Responsibility Law t periods ahead.

When applying this methodology to private firms, Merton (1974) uses the volatility of their assets as proxy for the stochastic component of assets evolution. We here employ as stochastic component (volatility) the sensitiveness of states' net current revenue to changes in the real GDP. We estimate the following panel to capture the sensitivity or elasticity of the net current revenue of each federated entity to variations of the national real GDP:[4]

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[4]Ideally, it would be better to use state-level real GDP. However, we use national-level GDP due to data unavailability.

$$\Delta NCR_{it} = \alpha_i + \eta_t + \sum_{i \in \mathscr{E}} \beta_i \cdot \Delta GDP_t + \gamma^T \cdot \text{varControl}_t + \mu_{it}, \qquad (5)$$

in which i indexes the Brazilian states, t represents time. Since the net current value of states also depend on other macroeconomic conditions of the country, we control for the exchange rate, inflation, and the interest rate in the vector varControl.

We extract the individual sensitivity or elasticity β_i of each state i, $i \in \mathscr{E}$, against changes in the real GDP ΔGDP_t. The state fixed effect α_i absorbs non-observed and time-independent regional particularities of the states.

In our empirical estimation, we use data from the 1st quarter of 2008 to the 1st quarter of 2017. $\mathscr{E}$ is the set of federated states in Brazil, including the Federal District. We estimate the specification (5) using clustered errors at the federative level and with fixed effects for time, η_t, and for federative entities, α_i.

We then estimate the volatility of state i, σ_{NCR}, for an expected variation of the real GDP t periods ahead as follows:

$$\sigma_{NCR}(\theta) = \beta_i \times \theta, \qquad (6)$$

in which $\theta \in \mathbb{R}$ is the expected variation of the real GDP t periods ahead. In this way, the probability of noncompliance of the states is conditional on the expected variation of the real GDP in the near future. Such conditioning can be useful to understand how the fiscal health of the states would suffer in a case of recession or expansion. Given that the noncompliance to the legal limits established by the Fiscal Responsibility Law can impose sanctions to the states, this would therefore cause real negative effects to the investments of the states.

The Fiscal Responsibility Law has several legal limits with which states must comply. Following the same reasoning as before, we can also estimate the probability of noncompliance of states to other components. In our methodology, we use two legal limits: (i) consolidated debt against net current revenue and (ii) personnel expenses against net current revenue. For completeness, we also derive the equations for the case of the personnel expenses limit in the following:

$$PD_{\text{Personnel}} = P\left(\frac{\text{Personnel}_t}{NCR_t} > 49\%\right)$$
$$= P\left(\text{Personnel}_t > 0.49 \cdot NCR_t\right)$$
$$= P\left(-d > \varepsilon\right)$$
$$= \mathcal{N}\left(-d\right), \tag{7}$$

in which $PN_{\text{Personnel}}$ is the probability of noncompliance of the personnel expenses with regard to the net current revenue t periods ahead of time.

The Fiscal Responsibility Law establishes as 60 % the limit of personnel expenses of the entire state (Executive, Legislative, Judiciary). Due to data unavailability, we only model the limit imposed to the Executive, which is 49 %.

Given that the sanctions fixed by the Fiscal Responsibility Law are applied whenever the states noncomply to a single legal limit, i.e. sanctions are applied whenever any of the boundaries of the consolidated debt or personnel expense are exceeded, we define the conditional probability of noncompliance of the states (PN) as the maximum between the probability of noncompliance regarding the consolidated debt and the personnel expenses:

$$PN = P\left(\text{FRL is violated} \mid \Delta PIB = \theta\right)$$
$$= \max\left[PN_{\text{Debt}}, PN_{\text{Personnel}}\right]. \tag{8}$$

Equation (8) shows that we measure probability of noncompliance in terms of the legal restriction that is performing worse. In addition, the probability of noncompliance is conditioned on the change of real GDP in t periods ahead. In this way, our measure could estimate the impact for the states in case of a deepening of the Brazilian crisis in the coming years. To do so, it would suffice to take a negative shock for the variation of real GDP, i.e. $\theta < 0$.

3.2 Contagion and amplification in the financial system

In order to estimate the financial contagion and thus quantify the potential systemic risk in the Brazilian financial system in case states default on their bank debt, we use Silva et al. (2017a)'s methodology, which we summarize below.

The model takes as input the losses that each bank would suffer from the credit default of states and evaluates to what extent such shock gets amplified through the financial exposures among banks (interbank market), among banks and firms (bank lending to the

real sector), and among firms (corporate trade network). To do so, we represent the financial and real sectors as a two-layer complex network. The financial sector layer includes banks that can expose themselves to each other through unsecured credit operations. The real sector layer contains the network of firms, in which supplier companies grant credit purchases to client companies and thus are exposed to the default of the latter. Exposures between economic agents—banks and/or firms—pave the way for a potential channel of contagion, and hence of amplification of losses.

Figure 2 displays the representative two-layer network of our economy, where the top and bottom layers make up the financial and real sector, respectively. A link from *A* to *B* indicates a financial vulnerability that originates either from the economic agent's asset or liability side of the balance sheet.
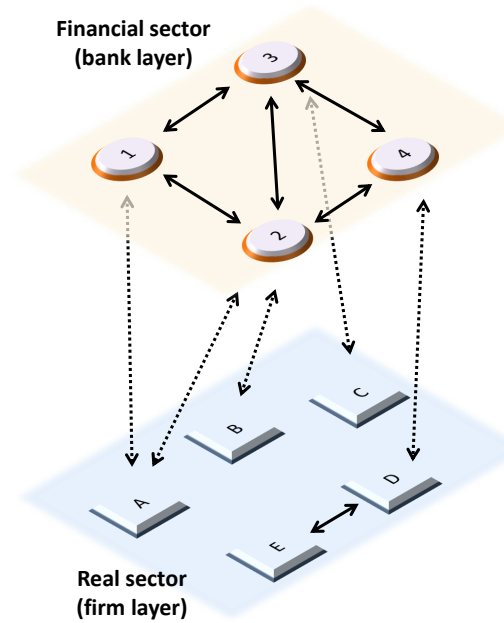
Figure 2: A financial network with two layers: banks (circle) and firms (squares). Links or edges represent financial vulnerability between two economic agents. We represent links that connect economic agents of the same nature with continuous lines and links that join the real and financial sectors with dashed lines.

In relation to the financial sector layer, banks interconnect to satisfy their liquidity needs and also to profit from financial operations in which they enjoy comparative advantage. For example, banks can group and use certain financial instruments to obtain mutual benefits with cost savings and increased profits. Banks that are competitive in loans to the nonfinancial sector but do not have the same ability to raise funds can borrow from banks with excess liquidity, thus obtaining the resources needed to provide credit to the nonfinancial sector. In the light of this association, both banks would be acting in their business in which they enjoy comparative advantage.

Therefore, on the asset side of the banks' balance sheets, links in the interbank market represent financial vulnerability due to unsecured investments. For example, bank

3 lends to bank 1 in Fig. 2 and therefore is subject to credit default. On the liability side, we model the funding risk that banks face to meet their short-term obligations. Whenever bank counterparties are under financial difficulties, we assume that they do not roll their short-term debts. As a result, borrowing banks may face liquidity problems if they do not have enough money to meet their short-term obligations. For example, the bank 1 has a funding vulnerability to the bank 3 if the exposure is due in the short term and the bank creditor 3 opts not to roll over that debt.

Linking the real and financial sector layers, banks and firm also interconnect to increase their profits. Firms are vulnerable to credit crunches from the financial sector. One the one hand, a sudden credit reduction may lead firms to readjust their project portfolios and thus their profits may be affected. In Fig. 2, the firm *A* is susceptible to credit restrictions from the bank 1, especially if its current debt matures in the short term. On the other hand, banks lend resources to firms and hence are subject to credit default. In Fig. 2, bank 1 lends to firm *A*, and therefore the former assumes the risk of a loan default.

There is a stress feedback between the real and financial sectors that is modeled through a financial accelerator. The methodology evaluates how the net worth of banks and firms deteriorates in view of an external shock. We model this external shock as a credit shock from the public sector. We assume that the federated states do not pay back their bank debt and hence banks write-off their assets to these entities. Consequently, their net worth reduces, which leads to the following spillover effects:

- *Real sector*: banks reduce credit to firms. Among other components, firms with liquidity problems will have to firesale their assets, thus potentially incurring losses as well. The decrease in the firms' net worth is observed by the creditor banks, which reprice down their loans to these firms. Consequently, banks again register further losses in their net worth.

- *Financial sector*: creditor banks also reprice down their investments to debtor banks that had loans to the public sector.

This dynamic process continues until the entire system reaches equilibrium to a unique fixed point. Silva et al. (2017a) demonstrate that the contagion mechanism between banks and firms with feedback between the financial and real sectors can be modeled with the following dynamic nonlinear system:

$$\mathbf{b}_i(t+1) = \min\left[1, \mathbf{b}_i(t) + \sum_{j \in \mathscr{B}} \mathbf{V}_{ij}^{(\text{bank-bank})} \Delta \mathbf{b}_j(t) + \sum_{u \in \mathscr{F}} \mathbf{V}_{iu}^{(\text{bank-firm})} \Delta \mathbf{f}_u(t)\right], \qquad (9)$$

$$\mathbf{f}_k(t+1) = \min\left[1, \mathbf{f}_k(t) + \sum_{u \in \mathscr{F}} \mathbf{V}_{ku}^{(\text{firm-firm})} \Delta \mathbf{f}_u(t) + \sum_{j \in \mathscr{B}} \mathbf{V}_{kj}^{(\text{firm-bank})} \Delta \mathbf{b}_j(t)\right], \qquad (10)$$

in which:

- $\mathbf{b}_i(t)$ e $\mathbf{f}_k(t)$ is the financial stress of bank i and firm k in iteration t.

- $\mathscr{B}$ e $\mathscr{F}$ are the sets of banks and firms, respectively.

- The matrices $\mathbf{V}_{ij}^{(\text{bank-bank})}$, $\mathbf{V}_{iu}^{(\text{bank-firm})}$, $\mathbf{V}^{(\text{firm-firm})}$ e $\mathbf{V}^{(\text{firm-bank})}$ are financial vulnerability matrices that model how shocks propagate in the network.

In our setup, we assume that the financial stress in all banks $i \in \mathscr{B}$ that have granted loans to federated entities is $\mathbf{b}_i(t) > 0$ and 0 otherwise.

4 Data

To evaluate the resilience of the financial system to the rising fiscal risk in Brazil, we consider a comprehensive set of government entities linked to the Executive, Legislative and Judicial branch of each Brazilian state. We also take into account state-owned companies whose regulation is government by public law. We define as public entities the set of all these government entities and state-owned companies.

In order to understand how the public entities can affect the stability of the financial system, we map all the outstanding credit operations carried out by these public entities using proprietary data from the Credit Bureau (SCR) of the Central Bank of Brazil. The SCR is a large-scale database that contains information on credit operations and securities with credit characteristics and respective guarantees contracted by individuals and legal entities from financial institutions in Brazil. Every financial institution in Brazil must report their credit operations with an outstanding amount greater than R\$ 200 (roughly US\$ 60) on a monthly basis, which reinforces the representativeness of our data set.

If a federated state does not pay its bank debts, the creditor banks suffer losses and thus reduce the net worth. The reduction of the net worth has implications for both the real and the financial sector. In the financial sector, banks reprice their investments toward banks that suffered loss and therefore register losses as well. To identify banks that may have these negative spillover effects, we use the interbank market that contains bilateral exposures among all banks in the Brazilian jurisdiction. We use data maintained by the Central Bank of Brazil, which is collected by several custodian agents in Brazil. For example, SCR is maintained by the Central Bank of Brazil and has credit operations among banks, while B3 is a private company that has information on private securities, swaps and options and must report these operations to the regulator.

To compose the Brazilian interbank market, and thus quantify the possible occurrence of financial contagion, we build the network of exposures between banks as follows.

We first extract from the custodian agents mentioned before all non-collateralized transactions between commercial, investment and development banks. Exposures encompass the classical interfinancial deposits among banks and also financial operations extracted from the B3. The contagion model also takes into account effects in the real sector, which is simplified, in this empirical exercise, only to publicly traded companies, responsible for a large part of the flow of payments between firms in Brazil.

Regarding the information on the fiscal situation of the states, we collect data from the National Treasury Secretariat, which receives and maintains accounting data received by public entities, through the Brazilian Public Sector Accounting and Tax Information System (Siconfi) and the Accounting Data Collection System (SISTN). The information in the annual balance sheet of entities is reported by the legal representatives of the entities in the format of the Annual Accounts Statement - CDA, or the Accounting Accounts Detail Chart (QCDC), both of which are approved by the National Treasury Secretariat.

In the following sections, we conduct an exploratory analysis of the public fiscal data collected at the National Treasury Secretariat and the proprietary credit data held by the Central Bank of Brazil.

4.1 Fiscal health of the federated entities

First, we analyze the evolution of important accounting variables for our analysis, which are the consolidated debt, net current revenue and personnel expenses of the states. Then we present the legal restrictions that the Brazilian Fiscal Responsibility Law imposes, which are: the ratio of consolidated debt to net current revenue and the ratio of personnel expenses to net current revenue.

4.1.1 Accounting variables

Consolidated debt. Figures 3a–3d show the consolidated debt of the Brazilian states from April 2008 to April 2017 for four representative state segments:

- *Very high consolidated debt*: Minas Gerais, Rio de Janeiro, Rio Grande do Sul and São Paulo.

- *High consolidated debt*: Alagoas, Bahia, Ceará, Goiás, Mato Grosso do Sul, Paraná, Pernambuco and Santa Catarina.

- *Moderate consolidated debt*: Amazonas, Federal District, Espírito Santo, Maranhão and Mato Grosso.

- *Low consolidated debt*: Acre, Amapá, Pará, Paraíba, Piauí, Rio Grande do Norte, Rondônia, Roraima, Sergipe, and Tocantins.

(a) Very high consolidated debt

(b) High consolidated debt

(c) Moderate consolidated debt

(d) Low consolidated debt
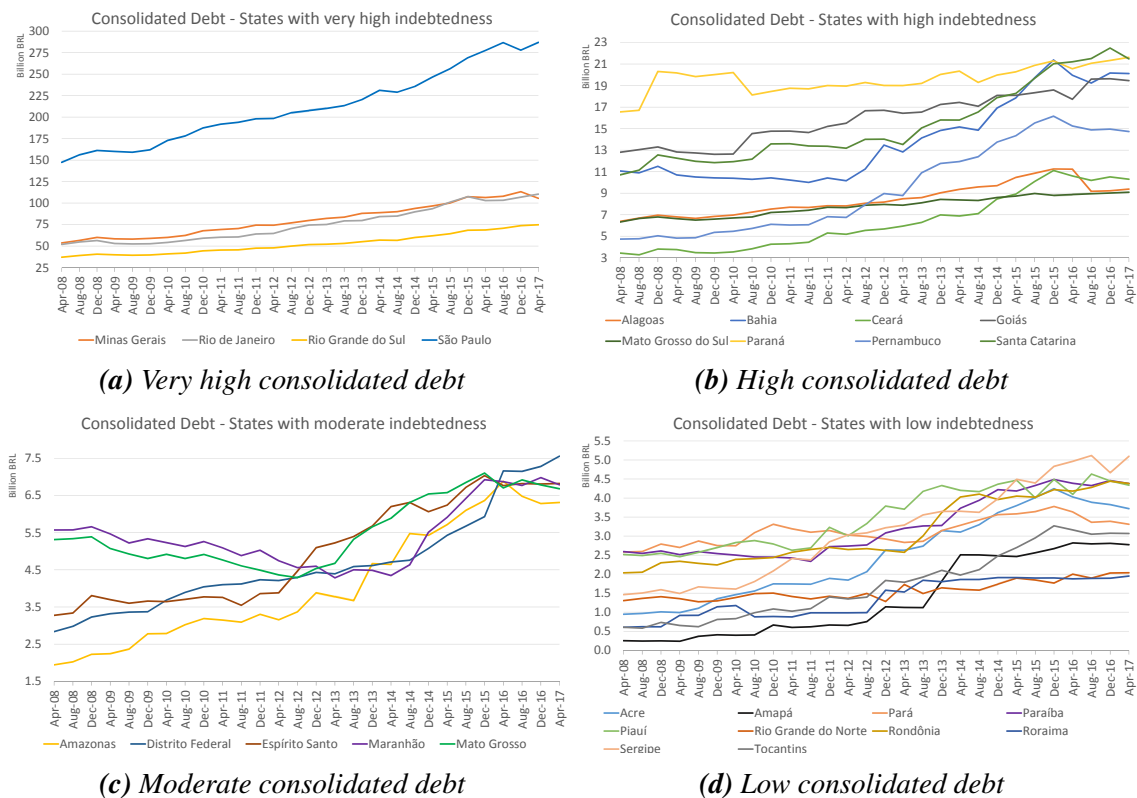
Figure 3: *Trajectories of the consolidated debt of the Brazilian states. We divide in four big state segments, depending on the level of the consolidated debt: (a) very high, (b) high, (c) moderate, and (d) low.*

In general terms, there is a growing total indebtedness in nominal terms of public entities,[5] especially for the state of São Paulo from August 2014 onwards. Minas Gerais maintained the second largest consolidated debt until December 2016, being surpassed by Rio de Janeiro after that date due to the rapid growth of Rio's debt. The consolidated debt of the state of Mato Grosso do Sul seems to grow much slower than its peers. In addition, the states of Maranhão and Mato Grosso were at a time of declining consolidated debt between April 2008 and mid-first half of 2013. After this period, consolidated debt rose faster than its peers.

Net current revenue. Figures 4a–4c depict the evolution of net current revenue of the Brazilian states. To facilitate the reading, we divide again into three state segments[6]:

- *High net current revenue*: Bahia, Minas Gerais, Paraná, Rio de Janeiro, Rio Grande do Sul, and São Paulo.

- *Moderate net current revenue*: Amazonas, Ceará, Federal District, Espírito Santo, Goiás, Maranhão, Mato Grosso, Pará, Pernambuco, and Santa Catarina.

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[5]Though the consolidated debt over GDP tends to decrease until the end of 2014 (Figure 1), the nominal consolidated debt of states constantly increases throughout the entire sample period (Figure 3) mostly due to inflation.

[6]We do not divide into four segment as before because many states have similar growth rates and levels.

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- *Low net current revenue*: Acre, Alagoas, Amapá, Mato Grosso do Sul, Paraíba, Piauí, Rio Grande do Norte, Rondônia, Roraima, Sergipe, and Tocatins.
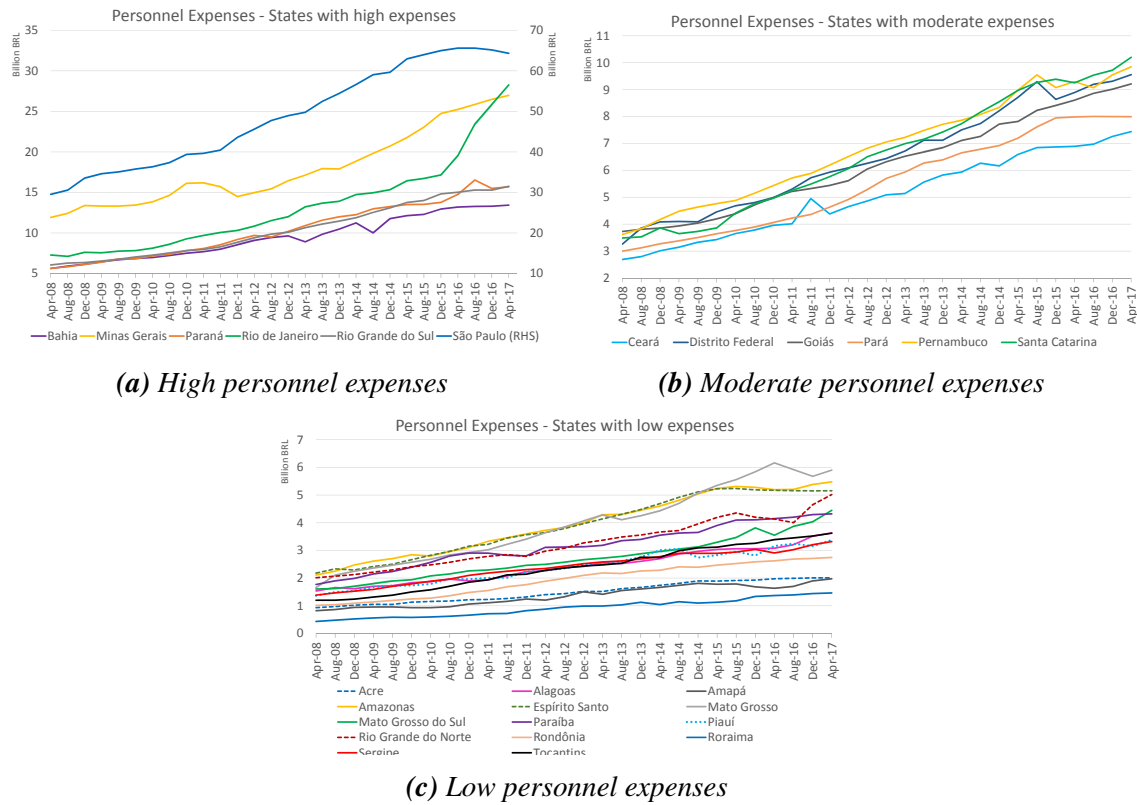

(a) High net current revenue

(b) Moderate net current revenue

(c) Low net current revenue
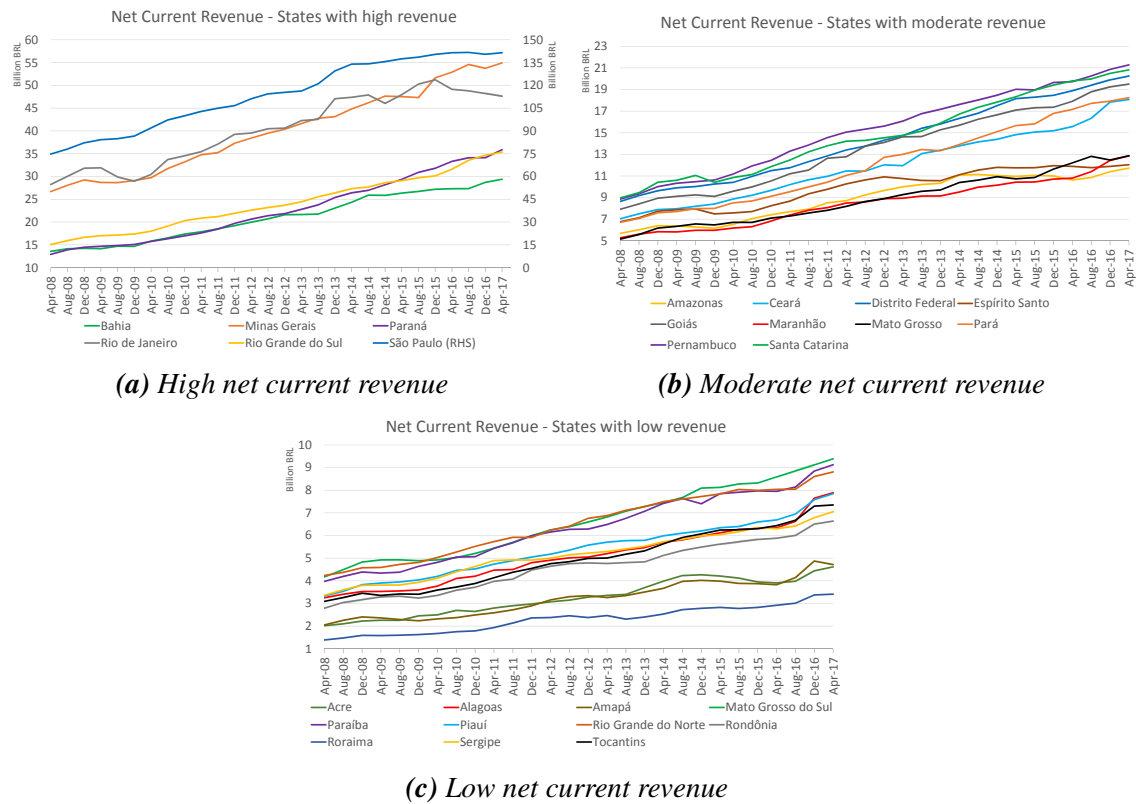
Figure 4: *Trajectories of the net current revenue of the Brazilian states. We divide in three state segments, depending on the level of the net current revenue: (a) high, (b) moderate, and (c) low.*

In general, there is an increase in net current revenue due to inflation and GDP growth. There is also a considerable decrease in the growth rate of São Paulo's net current revenue as from the beginning of 2014 and a decrease in Rio de Janeiro's net current revenue as from 2016.

Personnel expenses. Figures 5a–5b display the trajectory of the personnel expenses of the Brazilian states. We divide the exhibit into three state segments:

- *High personnel expenses*: Bahia, Minas Gerais, Paraná, Rio de Janeiro, Rio Grande do Sul, and São Paulo.

- *Moderate personnel expenses*: Ceará, Federal District, Goiás, Pará, Pernambuco, and Santa Catarina.

- *Low personnel expenses*: Acre, Alagoas, Amapá, Amazonas, Espírito Santo, Mato Grosso, Mato Grosso do Sul, Paraíba, Piauí, Rio Grande do Norte, Rondônia, Roraima, Sergipe, and Tocatins.

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(a) High personnel expenses

(b) Moderate personnel expenses

(c) Low personnel expenses

Figure 5: *Trajectories of the personnel expenses of the Brazilian states. We divide in three state segments, depending on the level of the personnel expenses: (a) high, (b) moderate, and (c) low.*

In general, there is an increase in personnel expenses of the Brazilian states. Contrary to that tendency, there is a decrease in personnel expenses in the state of São Paulo after 2016. Opposed to that, we can see a very rapid increase in personnel expenses in the state of Rio de Janeiro, also starting in 2016.

4.1.2 Legal indices imposed by the Fiscal Responsibility Law

Consolidated debt to net current revenue. Figure 6a illustrates the CD / NCR legal constraint for all Brazilian in January 2017. We note three state segments:

- *High CD / NCR*: Minas Gerais, Rio de Janeiro, Rio Grande do Sul, and São Paulo. Among these, the last three have already violated the 200 % legal limit that the Fiscal Responsibility Law imposes for states and therefore are subject to sanctions, including the impossibility of celebrating new credit operations.

- *Moderate CD / NCR*: Acre, Alagoas, Goiás, Mato Grosso do Sul, Santa Catarina, and Sergipe.

- *Low CD / NCR*: the remaining Brazilian states that have not been cited above, as well as the Federal District.

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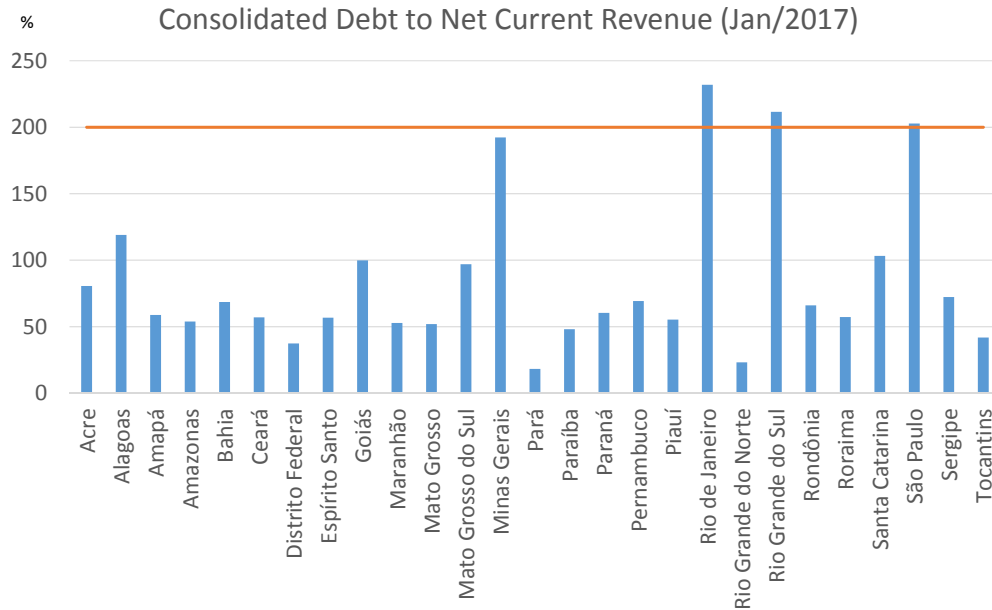
(a) Legal constraint CD / NCR in January 2017
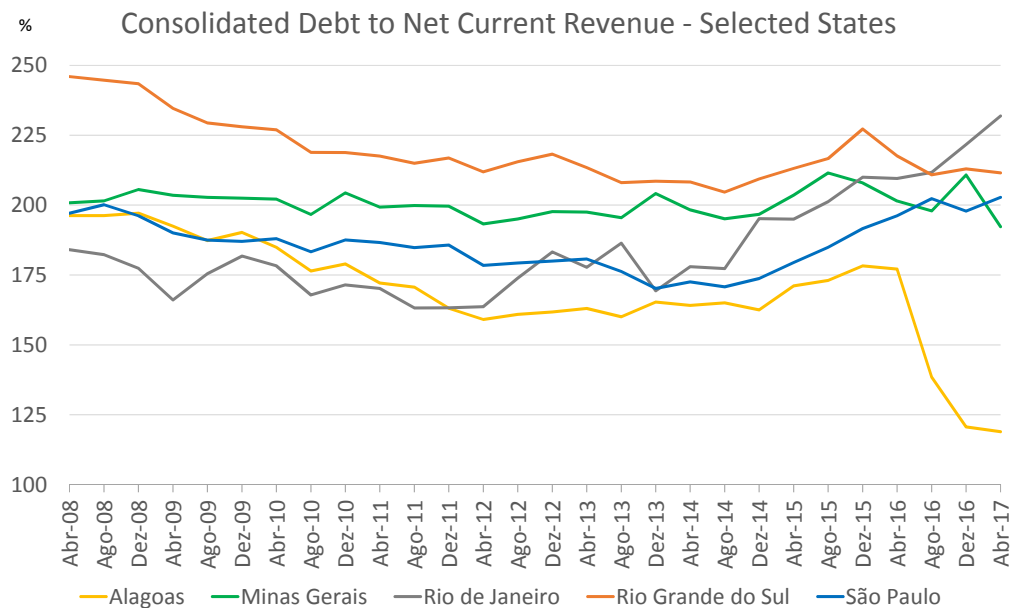
(b) Trajectory of the legal constraint CD / NCR for some states

Figure 6: *Fiscal situation of the Brazilian states regarding the legal restriction on the ratio consolidated debt to net current revenue. According to the Fiscal Responsibility Law, states must maintain this index under 200 %. If they do not comply with this legal restriction, they are subject to sanctions, including the impossibility of celebrating new credit operations.*

Figure 6b displays the trajectory of the legal constraint CD / NCR of the states of Alagoas, Minas Gerais, Rio de Janeiro, Rio Grande do Sul, and São Paulo from 2008 to 2017. We see that:

- Rio Grande do Sul have always remained above the legal limit of 200 % that the Fiscal Resposibility Law establishes for all Brazilian states. Nonetheless, such state

shows a decreasing trend in the period, going from 250 in April 2008 to 210 in April 2017, which suggests a slight improvement on its consolidated debt in relation to the net current revenue.

- Minas Gerais has remained practically steady in the period, with levels below the legal threshold of 200 %.

- Rio de Janeiro had a considerable increase, going from 180 in April 2008 (below the legal threshold) to 230 in April 2017 (well above the legal threshold). This fact indicates a deep and sudden worsening of the consolidated debt to the net current value in the present days.

- São Paulo begins the period in compliance with the legal threshold, decreases to a minimum of 170 in April 2014, but then the legal index again rises rapidly to the legal threshold of 200 % in April 2017.

- Alagoas has steadily decreased its index, notably after April 2016, with a 50-point drop in a single year.
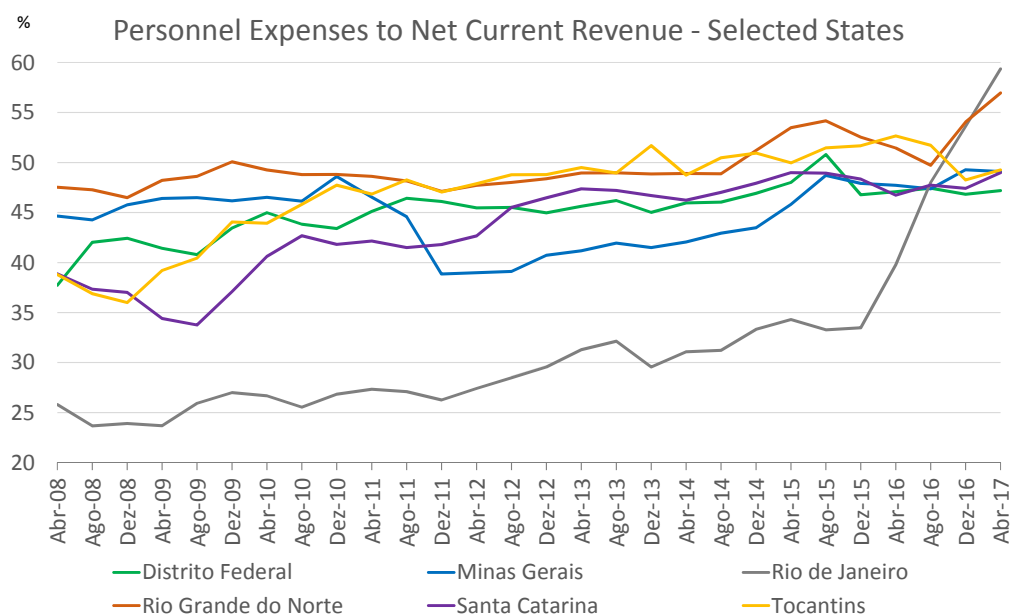
Personnel expenses to net current revenue. Figure 7a illustrates the personnel expenses / NCR legal ratio for all Brazilian states in January 2017. Due to data unavailability, we only plot the personnel expenses from the Executive. The Fiscal Responsibility Law disciplines that states must have an index personnel expenses / NCR below 49 % for the Executive Branch. We observe that almost all states are facing budget problems with personnel expenses relative to their respective net current revenues. In contrast, only the richest Brazilian states are having problems to comply with the CD / NCR legal index. Some states, such as Rio de Janeiro and Rio Grande do Sul and Tocatins, have exceeded the legal threshold in January 2017 and therefore are not complying with the Fiscal Responsibility Law.

Figure 7b shows the evolution of the personnel expenses / NCR index for the states of Minas Gerais, Rio de Janeiro, Rio Grande do Norte, Santa Catarina, and Tocantins. We note that:

- The Federal District and the states of Minas Gerais and Rio Grande do Sul have always suffered to meet the legal restriction on personnel expenses fixed by the Fiscal Responsibility Law. In particular, there is a vertiginous growth for the state of Rio Grande do Sul in the second half of 2016 onwards.

- The states of Santa Catarina and Rio de Janeiro show compliant indexes for personnel expenses legal restriction in 2008, especially the state of Rio de Janeiro with a low index of 26. However, the personnel expenses index grows at a very fast pace, especially after 2016.
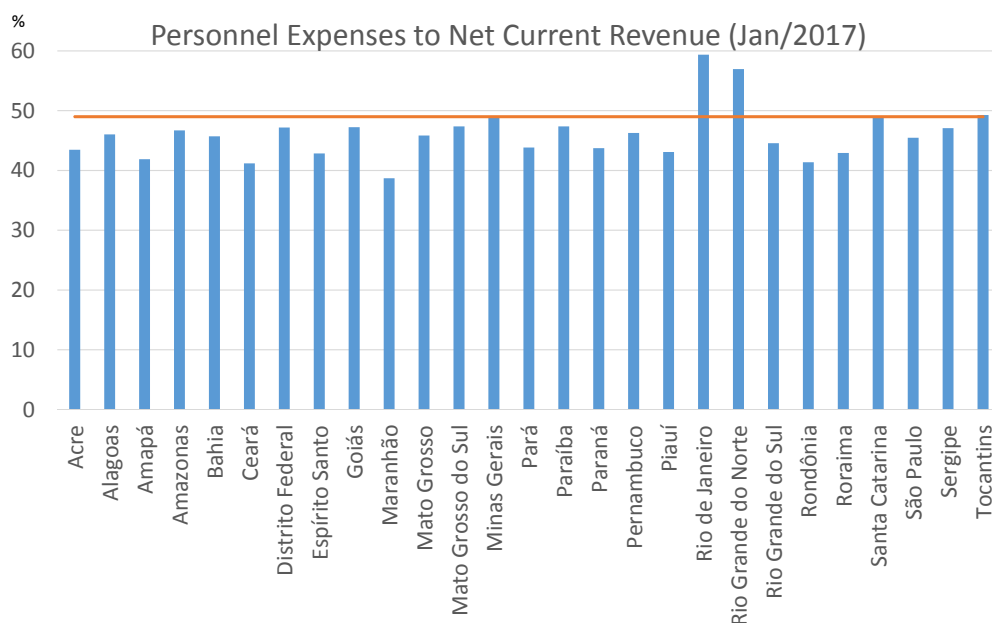
(a) Legal constraint personnel expenses / NCR (Executive) in January 2017

(b) Trajectory of the legal constraint personnel expenses / NCR (Executive) for some states

Figure 7: *Fiscal situation of the Brazilian states regarding the legal restriction on the ratio personnel expenses to net current revenue. According to the Fiscal Responsibility Law, states must maintain this index under 49 % for the Executive Branch. If they do not comply with this legal restriction, they are subject to sanctions, including the impossibility of celebrating new credit operations.*

Most critical (or binding) legal restrictions to states. Table 1 reports which of the two legal indicators fixed by the Fiscal Responsibility Law is more critical (or binding) for the states, i.e., more difficult to meet. We note that, once a single legal restriction is violated, the state is already subject to sanctions such as the impossibility of celebrating credit operations or receiving voluntary transfers from the federated government. There-

fore, we look at the fiscal health of states by the legal restriction that is performing the worst.

Of the 27 federated entities, 5 (18.5 %) are more critical in the CD / NCR index, while 22 (81.5 %) are more critical in the NCR / Personal Expense Index. The richest states in Brazil tend to have greater difficulty in meeting the constraint related to consolidated debt, while medium and small states, in general, that related to personnel expenses.

Table 1: *Most critical (or binding) legal restriction that the Fiscal Responsibility Law (FRL) imposes on states: (i) ratio consolidated debt to net current revenue or (ii) ratio personnel expenses on the Executive Branch to net current revenue. Of the 27 federated entities, 5 (18.5 %) are more critical in the CD / NCR index, while 22 (81.5 %) are more critical in the personnel expenses / NCR index.*

State	Worst FRL criterion	State	Worst FRL criterion
Acre	Personnel expenses / NCR	Paraíba	Personnel expenses / NCR
Alagoas	CD / NCR	Paraná	Personnel expenses / NCR
Amapá	Personnel expenses / NCR	Pernambuco	Personnel expenses / NCR
Amazonas	Personnel expenses / NCR	Piauí	Personnel expenses / NCR
Bahia	Personnel expenses / NCR	Rio de Janeiro	CD / NCR
Ceará	Personnel expenses / NCR	Rio Grande do Norte	Personnel expenses / NCR
Federal District	Personnel expenses / NCR	Rio Grande do Sul	CD / NCR
Espírito Santo	Personnel expenses / NCR	Rondônia	Personnel expenses / NCR
Goiás	Personnel expenses / NCR	Roraima	Personnel expenses / NCR
Maranhão	Personnel expenses / NCR	Santa Catarina	Personnel expenses / NCR
Mato Grosso	Personnel expenses / NCR	São Paulo	CD / NCR
Mato Grosso do Sul	Personnel expenses / NCR	Sergipe	Personnel expenses / NCR
Minas Gerais	CD / NCR	Tocantins	Personnel expenses / NCR
Pará	Personnel expenses / NCR		

4.2   Bank credit to federated states

To improve the visualization of our results, we only report the bank credit portfolio of the four states with the largest internal contractual debt in the financial system, which are[7]: Rio de Janeiro, São Paulo, Minas Gerais, and Goiás. In addition, we include Rio Grande do Sul because it is experiencing fiscal difficulties.[8]

The funding portfolio of Brazilian states comes in great part from refinacing operations with the federal government and bank credit. Before understanding how states can impact the financial system through a potential default of their bank loans, we first report in Table 2 the amount of outstanding debt of states to banks and to the federal government in December 2015 and June 2016. In this way, we can understand the representativeness of each type of funding to Brazilian states.

The most representative creditor of these five states is the federal government, with exception for the state of Goiás, whose debt is roughly equally divided between the federal government and banks. Thus, the indebtedness of the states is largely to the federal

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[7]In decreasing order of indebtedness, taking as reference data from June 2016.

[8]According to the government, states that are experiencing fiscal difficulties are: Rio de Janeiro, Minas Gerais and Rio Grande do Sul. While the first two are among the most indebted ones in the financial system, Rio Grande do Sul does not have large amounts of loans.

| | | | | | | In billions R$ |
| | December 2015 | | | June 2016 | | |
State	Government	Bank	Total	Government	Bank	Total
Goiás	9.50	7.63	17.12	8.98	7.13	16.11
Minas Gerais	78.88	9.45	88.33	83.04	8.44	91.48
Rio de Janeiro	56.34	22.39	78.73	59.48	21.18	80.66
Rio Grande do Sul	51.91	1.67	53.58	54.72	1.65	56.37
São Paulo	220.14	14.57	234.71	232.77	14.35	247.12

government and not the banking system. Nonetheless, it stands as a relevant issue to study the effects of a potential default of the bank debt to the financial system, because banks are highly connected to other markets and therefore have the potential to amplify shocks.

There are other indirect channels of shock transmission to the financial system through the federated states that we are abstracting away in this paper. For example, the nonpayment to civil servants from a specific state would lead to an increase in the delinquency rate of the bank loans granted to these employees. In addition, government indebtedness can force a reduction of investments and therefore affect private firms in the form of reduced demand and incentives, which in turn could again hit the financial sector through further loan defaults.

In order to understand which banks are more susceptible to defaults from the public entities, we evaluate the outstanding bank credit in terms of the bank control, which we segregate as state-owned and private (domestic and foreign) banks. Table 3 discriminates the outstanding bank credit in terms of bank control, which can be state-owned, domestic private, and foreign private banks. The great majority of bank credit that Brazilian states take come from state-owned banks. We highlight that the state of Rio de Janeiro holds a small portion of bank credit from private banks.

| | December 2015 | | June 2016 | |
State	State-owned banks	Private banks	State-owned banks	Private banks
Goiás	**100.00%**	0.00%	**100.00%%**	0.00%
Minas Gerais	**100.00%**	0.00%	**99.60%**	0.40%
Rio de Janeiro	**94.60%**	5.40%	**93.70%**	6.30%
Rio Grande do Sul	**100.00%**	0.00%	**100.00%**	0.00%
São Paulo	**100.00%**	0.00%	**100.00%**	0.00%

Figure 8 portrays the ratio of the total outstanding credit that banks grant to federated states to the total capitalization of the corresponding creditor banks. Given that banks

reflect losses of credit to the public entities as a reduction of their capital, this ratio gives us a hint on how banks would withstand losses coming from the public entities. In case losses surpass bank capital, then defaults take place.
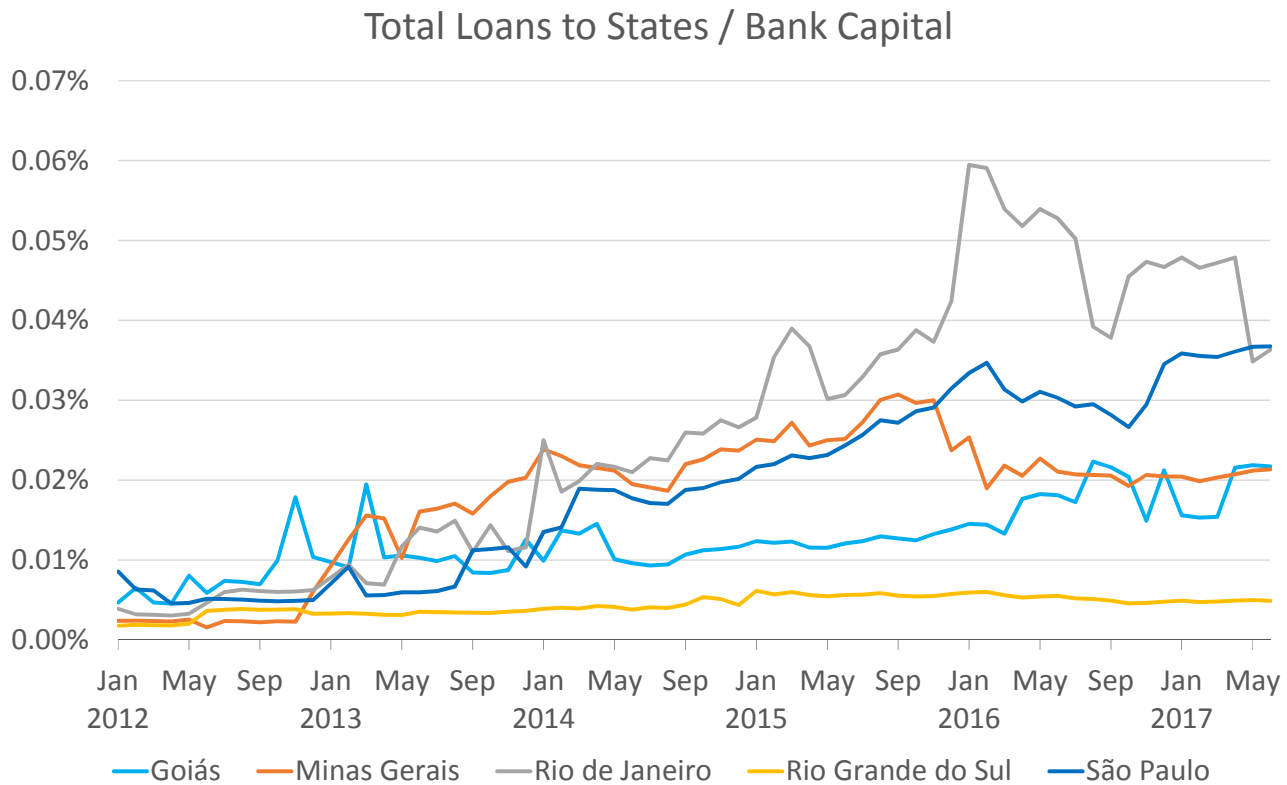
Figure 8: *Ratio of the states' outstanding bank credit to the total capitalization of corresponding creditor banks. Given that banks reflect losses of credit to the public entities as a reduction of their capital, this ratio gives us a hint on how banks would withstand losses coming from the public entities. In case losses surpass bank capital, then defaults take place.*

5 Results

In this section, we present the main results of this work. We start with the estimation of the probability of noncompliance of states in a hypothetical scenario that simulates a deepening of the recession in Brazil. We then analyze the impact that the financial system would suffer in the event that states do not honor their bank debt. Finally, we combine both the probability of noncompliance and the impact in the financial system to compose a statistical indicator of expected loss in the financial system.

5.1 Estimation of the probability of noncompliance of federated states in Brazil under a stressed scenario

In this section, we estimate the probability of noncompliance of Brazilian states using a hypothetical scenario in which the Brazilian real GDP decreases by 5%.

Estimating the probability of noncompliance of states is a useful tool to classify, in a prospective way, the fiscal health of these entities and hence measure fiscal risk. This prospective estimation gains even more importance in the current scenario of rising debt and revenue reduction that Brazil is facing, which raises concerns about the sustainability of the fiscal health of states.

We evaluate the probability of noncompliance of states according to the legal restrictions CD / NCR and personnel expenses / NCR for a one-year horizon. We assume a hypothetical deepening of the recession in Brazil with a projection of real GDP decrease of 5 % in the next year. The expected negative change in real GDP for the next year generates a decrease in net current revenues of the states, which we estimate using the empirical specification in Section 3.1.

Elasticity of net current revenue of states to changes in the real GDP. Table 4 reports our estimates for the elasticities of the net current revenue to the real GDP. In our exercise, we simulate a hypothetical decrease in the real GDP of 5 % and then evaluate, using these elasticities, how the net current revenue of each Brazilian state changes. Finally, we use the stressed net current revenue to estimate the probability of noncompliance of states in a one-year horizon. In the Table 4, for example, São Paulo would have a projected reduction of 0.04 * 5 = 0.20 % of net current revenue for the coming year as a result of a hypothetical decrease of 5% in the real GDP, given that the consolidated debt and personnel expenses would increase according to inflation.

Table 4: *Annual elasticity of the net current revenue (NCR) relative to real GDP. For example, a 1 % increase in real GDP over the previous period would cause NCR of the state of São Paulo to increase by 0.04 % in the following year. We indicate statistical significance by means of the p-value: *** corresponds to a p-value of less than 1%, **, at a p-value between 1% and 5% and *, at a p-value between 5% and 10%.*

State	Elasticity NCR-PIB	Estado	Elasticity NCR-PIB
Acre	0.12***	Paraíba	0.08***
Alagoas	0.06***	Paraná	0.02*
Amapá	0.20***	Pernambuco	0.04***
Amazonas	0.13***	Piauí	0.06***
Bahia	0.06***	Rio de Janeiro	0.09***
Ceará	0.11***	Rio Grande do Norte	0.08***
Federal District	0.10***	Rio Grande do Sul	0.07***
Espírito Santo	0.03**	Rondônia	0.00
Goiás	0.03***	Roraima	0.08***
Maranhão	0.06***	Santa Catarina	-0.01
Mato Grosso	0.05***	São Paulo	0.04***
Mato Grosso do Sul	0.05***	Sergipe	0.11***
Minas Gerais	0.11***	Tocantins	0.09***
Pará	-0.02		

We observe a great heterogeneity in the sensitivity or elasticity of the net current income of the states in relation to real GDP. The most sensitive state is Amapá, followed by Amazonas, Acre, Minas Gerais, Sergipe and Rio de Janeiro. We consider that the net current revenue is inelastic for cases in which the coefficient is statistically non-significant, such as for the states of Pará, Rondônia and Santa Catarina.
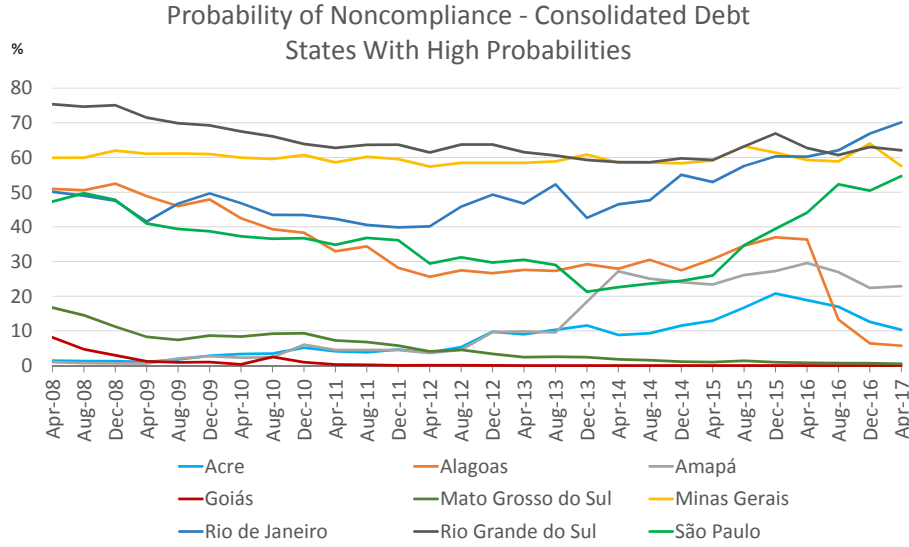
Probability of noncompliance for the legal constraint CD / NCR. Figures 9a–9b show our estimates of the states' probability of noncompliance in the next year for the legal restriction CD / NCR fixed by the Fiscal Responsibility Law. For each point in the figures, we evaluate the probability of noncompliance in the upcoming year in an independent manner. For instance, with regard to the point in the figure referring to April 2014 for the state of São Paulo, we evaluate its probability of noncompliance until April 2015.

The probability of noncompliance for the legal constraint CD / NCR of the state of Rio de Janeiro rapidly increases after 2014, surpassing that of the state of Rio Grande do Sul by April 2016, which was the state with the highest probability of noncompliance until then. Minas Gerais also presents a high probability of noncompliance, but without positive or negative trends. The state of Mato Grosso do Sul also shows a large increase in the probability of noncompliance for the CD / NCR legal constraint in August 2015, reaching the fourth place until the end of the sample, April 2017. Unlike the other states, the state of Alagoas shows a consistent decrease in the probability of noncompliance, especially from 2016.
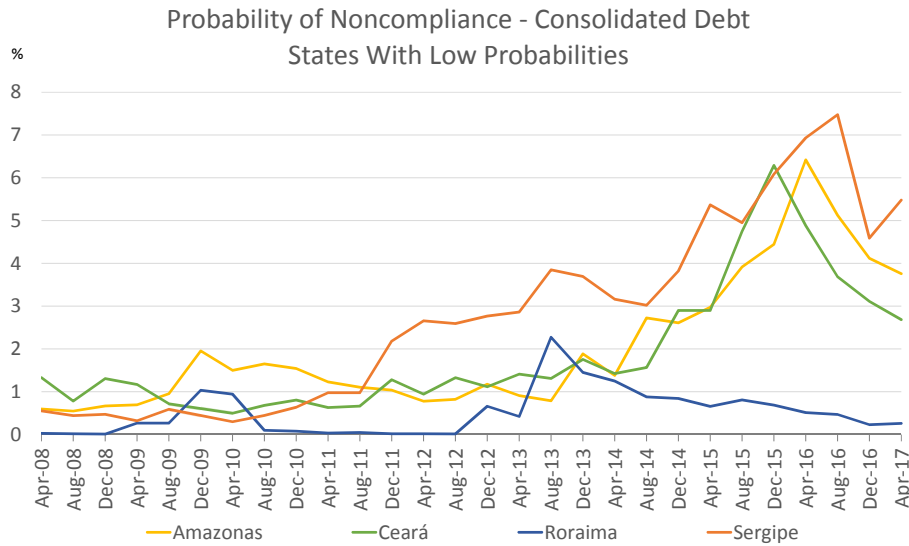
Probability of noncompliance for the legal constraint personnel expenses / NCR. Figures 10a–10b display our estimates for the probability of noncompliance of Brazilian states in the next year. In general, there is an increase in the probability of noncompliance of the personnel expenses / NCR for all states after 2013.

Among the states with the highest probability of noncompliance, Rio de Janeiro stands out due to the very rapid increase after 2016. By the end of 2015, such state had a probability of noncompliance nearing 25%. After that point, it increased up to 60% in April 2017. The Federal District also shows a strong increase in the probability of noncompliance since 2008. The state of Tocantins also presents a significative increase in the probability of noncompliance, but only after 2009. The state of Amapá presents a stable, though high, probability of noncompliance for the legal constraint personnel expenses / NCR relatively to other federated entities. However, it starts to decrease in 2014 onwards.

The increasing pattern of probability of noncompliance in personnel expenses / NCR also occurs in those states with low values of this indicator. Among this subset of states, we highlight the significant increase of this probability in the states of Mato Grosso and Pernambuco, reaching 50% and 40%, respectively, in the beginning of 2017.
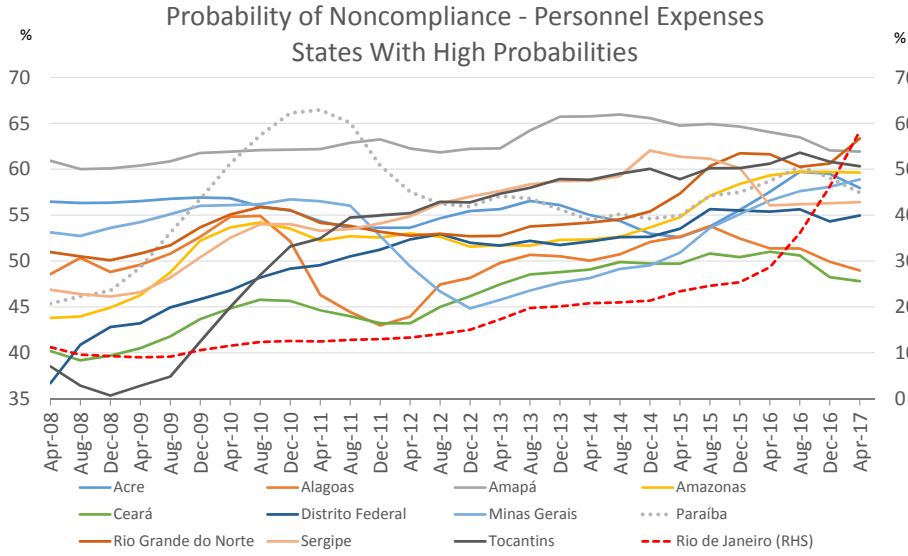
(a) *High probability of noncompliance for the legal restriction CD / NCR in the next year*
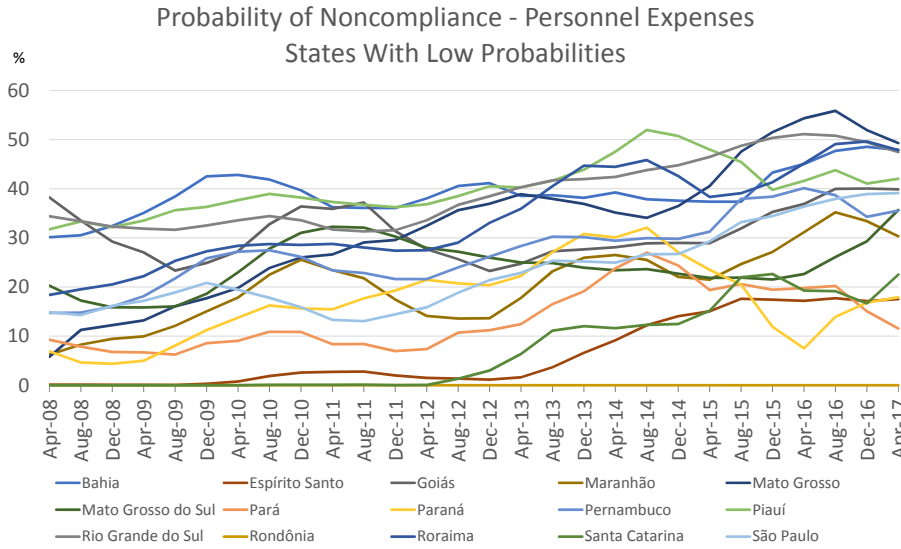

(b) *Low probability of noncompliance for the legal restriction CD / NCR in the next year*

Figure 9: *Trajectory of the probability of noncompliance for the legal restriction CD / NCR index in the next year for the Brazilian states. For each point in the figures, we evaluate the probability of noncompliance in the upcoming year in an independent manner. For instance, with regard to the point in the figure referring to April 2014 for the state of São Paulo, we evaluate its probability of noncompliance until April 2015. We divide the estimates in accordance with the level of the probability of noncompliance: (a) high and (b) low. We do not plot the curves for states that have probability of noncompliance close to zero.*

The states of Espírito Santo and Santa Catarina also present an increase in the probability of noncompliance, but at levels still low compared to other states. The volatility of the probability of noncompliance of states with low values is higher than those with higher values.

(a) High probability of noncompliance for the legal constraint personnel expenses / NCR in the next year.

(b) Low probability of noncompliance for the legal constraint personnel expenses / NCR in the next year

Figure 10: *Trajectory of the probability of noncompliance for the legal restriction personnel expenses / NCR index in the next year for the Brazilian states. For each point in the figures, we evaluate the probability of noncompliance in the upcoming year in an independent manner. For instance, with regard to the point in the figure referring to April 2014 for the state of São Paulo, we evaluate its probability of noncompliance until April 2015. We divide the estimates in accordance with the level of the probability of noncompliance: (a) high and (b) low. We do not plot the curves for states that have probability of noncompliance close to zero.*

5.2   Impact of the federated entities on the financial system

In this section, we quantify the resilience of the Brazilian financial system in the event that federated entities default on their bank debt. Although bank debt is not the most representative debt of Brazilian states, we should carefully look at these exposures

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because the financial system is a backbone through which several markets communicate. Therefore, localized shocks can propagate through the financial interlinkages and potentially develop into widespread and systemic events.

We first estimate the rate at which the financial system amplifies losses in the event that states do not honor their bank debt, disregarding the probability of occurrence of such event. This empirical exercise is important for public authorities in the sense that they provide estimates as to how losses would amplify in case a specific state defaults.

Figure 11 shows the short-term consequences of credit default that the states of Rio de Janeiro, Minas Gerais, Rio Grande do Sul, São Paulo, and Goiás would impose on the financial system in terms of the capitalization of the entire financial system. We measure the overall impact in terms of the banking system's total capital loss. In addition to this direct hit, we evaluate the negative spill-over effects due to potential contagion that would arise due to unsecured exposures inside the interbank market and bank credit to the real sector. We select these five states because they are the most fragile in terms of public finances and are nearing non-compliance of the Fiscal Responsibility Law.

The impact of credit defaults of the state of Rio de Janeiro on the financial system significantly increases until the end of 2015. After this date, the outstanding bank credit starts to slowly decrease while creditor banks of the Rio de Janeiro state increase capitalization. Both factors lead to a decrease of the credit default impact of Rio de Janeiro on the financial system. São Paulo has a similar pattern. However, unlike Rio de Janeiro, potential contagion starts to take significant amounts only after September 2013, when São Paulo made new loans in the financial system. The parcel due to potential contagion is the largest for credit defaults from the state of Minas Gerais. This fact is partly because their creditor banks are relatively less capitalized, which increases the vulnerability and propensity of shock propagation inside the network.

Overall, in the short term, the Brazilian financial system is resilient to shocks from the analyzed public entities in the sense that no banks would go bankrupt. We attribute this factor to the high levels of bank capital in the financial system, which serves as a shock barrier to external negative shocks. Nonetheless, we see a significant downfall of the financial system capitalization, which could have further negative consequences for the real sector in the medium and long run that our systemic risk model cannot capture because it assumes the network structure as exogenous.

Table 5 segregates the total impact on the financial system—comprising the initial shock and the potential contagion—in terms of bank control: state-owned, domestic private, and foreign private banks. The credit defaults from the five states that are facing the heaviest fiscal problems would cause the largest losses in state-owned banks. These banks not only are the major creditor banks to these states but also take central positions in the network, which make them susceptible to receiving shocks in the form of financial

(a) Rio de Janeiro

(b) São Paulo

(c) Goiás

(d) Minas Gerais

(e) Rio Grande do Sul
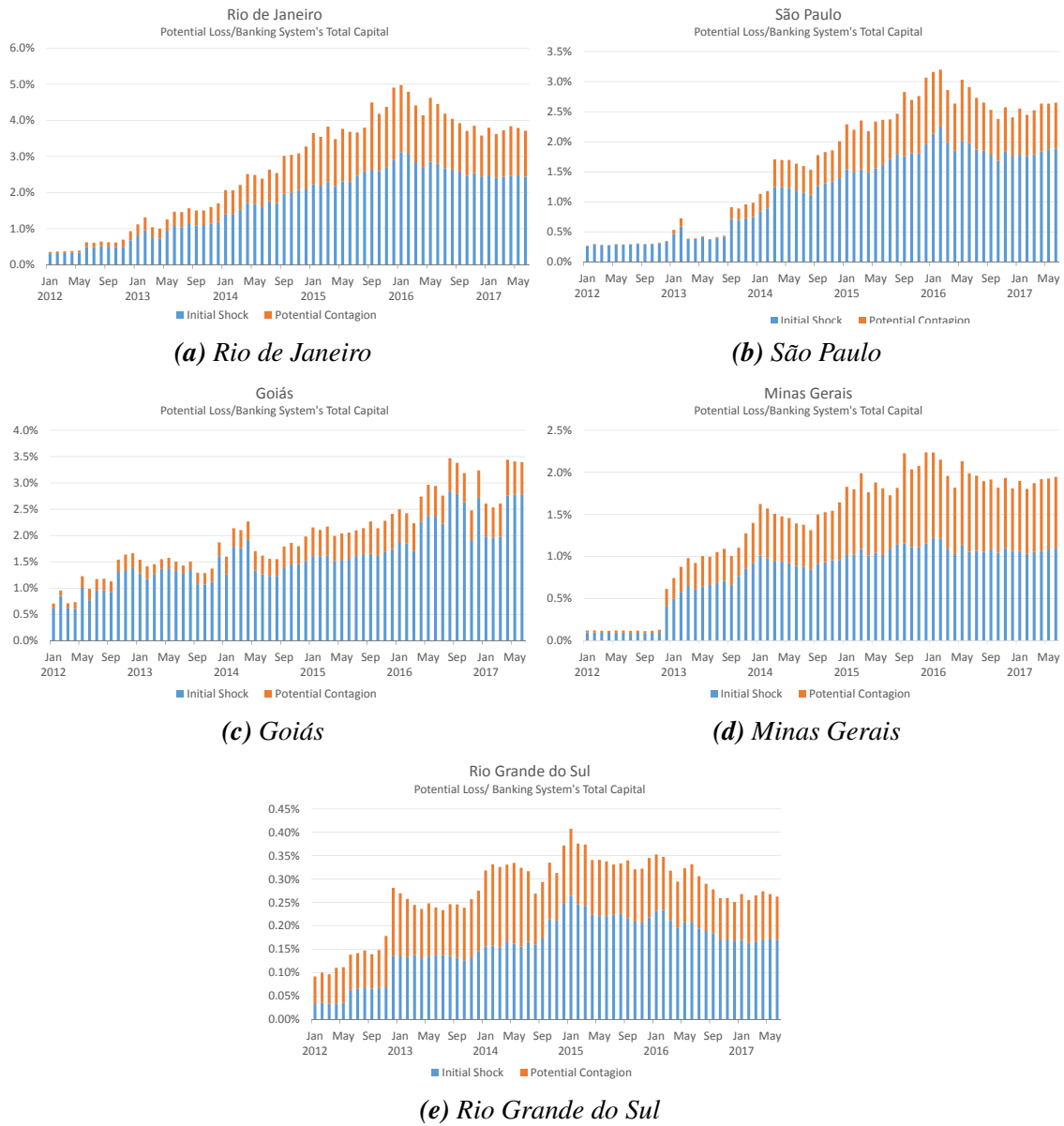
Figure 11: Initial shock and the corresponding potential contagion to which the financial system is subject. We measure the overall impact in terms of the banking system's total capital. The initial shock component represents the losses that the financial system would register if federated states did not pay back their bank debt. The potential contagion component indicates potential losses due to financial contagion and amplification (negative spill-over inside the network). We evaluate the initial shock and the corresponding potential contagion for each point in time in an independent manner for the states of (a) Rio de Janeiro, (b) São Paulo, (c) Goiás, (d) Minas Gerais, and (e) Rio Grande do Sul.

contagion. Both factors contribute to increasing financial stress over these state-owned banks. Domestic private banks would be the second most affected bank control category, followed by foreign private banks.

Table 5: *Proportion of losses that banks take due to credit defaults from federated entities in December 2015 and 2016, broken down by the bank control: state-owned and private (domestic and foreign) banks. We only report results for the five states that are facing the heaviest fiscal problems at the moment: Rio de Janeiro, São Paulo, Goiás, Minas Gerais, Rio Grande do Sul. We highlight in bold the type of bank control that suffers the most at each time point and federated state.*

	December 2015		June 2016	
State	**State-owned banks**	**Private banks**	**State-owned banks**	**Private banks**
Goiás	**95.31**%	4.69%	**97.08**%	2.92%
Minas Gerais	**97.00**%	3.00%	**98.62**%	1.38%
Rio de Janeiro	**96.53**%	3.47%	**98.19**%	1.81%
Rio Grande do Sul	**96.15**%	3.85%	**98.10**%	1.90%
São Paulo	**96.02**%	3.98%	**97.65**%	2.35%

5.3 Expected impact of the federated entities on the financial system
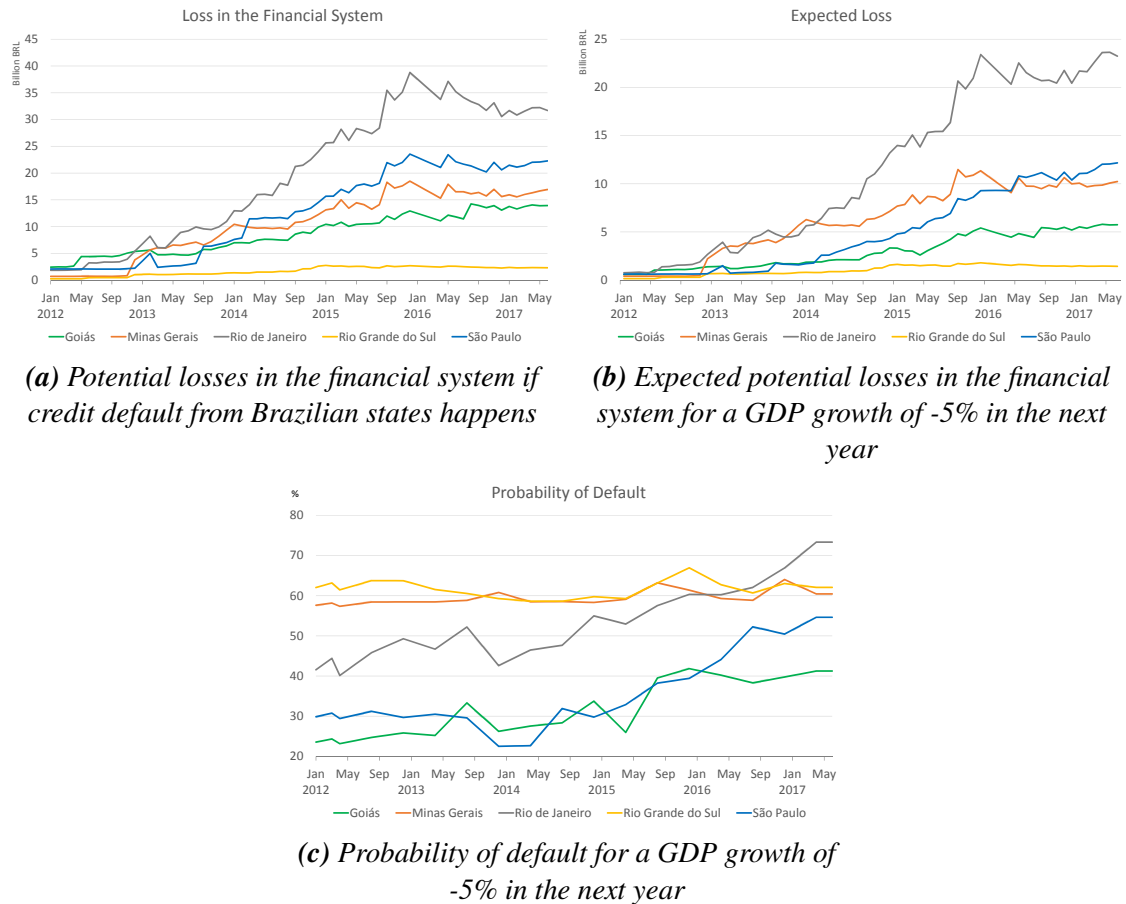
In this section, we weigh the potential loss that the financial system would suffer if a federated entity default on its bank loans with the probability of occurrence of this event. This weighting rules out events that would impose large in the losses in the system but with negligible probability of occurrence.

As discussed before, we assume that federated entities that do not comply with the legal constraints imposed by the Fiscal Responsibility Law are undergoing serious fiscal problems, preventing them to fully pay back their bank loans. We take the conservative approach here in terms of systemic risk analysis and assume that federated entities do not pay back their remaining outstanding bank debt. We estimate a state's probability of credit default as the highest probability of noncompliance between the legal constraints consolidated debt / net current revenue and personnel expenses / net current revenue. This is because the noncompliance of a single legal constraint already makes the state subject to legal sanctions.

Figure 12a shows the potential losses that the financial system would suffer from credit defaults from the states of Rio de Janeiro, São Paulo, Goiás, Minas Gerais, and Rio Grande do Sul. We report potential monetary losses, which are the sum of losses due to the credit default and the potential financial contagion that could happen inside the network. Figure 12c indicates the probability of default of these bank loans per each state, which we estimate in accordance with the highest probability of noncompliance of the legal restrictions regarding consolidated debt and personnel expenses. In the previous section, we have verified that more than 80 % of the Brazilian states have difficulties to comply with the personnel expenses legal constraint. However, the most indebted states, such as Rio de Janeiro, São Paulo, Minas Gerais, and Rio Grande do Sul, are exceptions and are struggling to comply with the legal constraint on consolidated debt.

Figure 12b portrays the expected loss in the financial system, which we evaluate as a point-to-point multiplication of Figures 12a and 12c. The expected losses from the most fragile states, on average, decrease from 2016 onwards, which may be related to

the federal government's agreements to the states and Federal District. Among these agreements, states can temporarily suspend payments to the Treasury, which, in theory, would give then surplus cash to pay back their bank debt and other obligations.

(a) Potential losses in the financial system if credit default from Brazilian states happens

(b) Expected potential losses in the financial system for a GDP growth of -5% in the next year

(c) Probability of default for a GDP growth of -5% in the next year

Figure 12: Comparison between (a) potential losses in the financial system if credit default from Brazilian states happens and (b) the respective expected loss, in which we weigh the conditional loss by its probability of occurrence (probability of default evaluated for a GDP growth of -5% in the next year). (c) Probability of default of the Brazilian states. We calculate the graph from item (b) by a point-to-point multiplication of the graphs from items (a) and (c).

In 2012, Goiás is the state that would inflict the largest losses to the Brazilian financial system. However, the occurrence of this event of credit default is more unlikely comparatively to the state of Rio de Janeiro in the same year. In this way, the combination of the impact and probability yields a smaller expected loss to the financial system for the state of Goiás in relation to the state of Rio de Janeiro in the same year. From 2013 onwards, Rio de Janeiro rapidly becomes the most harmful Brazilian state to the financial system, both unconditionally and conditionally. However, if we bring in the probability of occurrence of the credit default, the potential expected losses of Rio de Janeiro would significantly drop 40% in relation to its unconditional losses.

If we do not consider the probability of occurrence of defaults, São Paulo would be the second state most harmful state to the Brazilian financial system from March 2014,

followed by Minas Gerais. However, the expected losses to the financial system due to São Paulo are smaller than those from Minas Geiras until March 2016, due to its comparatively stronger fiscal health (Figure 12c).

6 Conclusions

This work collaborates with the literature of public finance and banking in two important points. First, we develop a new methodology to estimate the probability of noncompliance of Brazilian states to a legal framework. We innovate in the sense that the methodology not only uses historical data but also embodies the expectation of evolution of the real GDP. Second, we use a comprehensive systemic risk model that encompasses the real and financial sector to quantify and understand the resilience of the financial system to credit defaults from Brazilian states.

We find that banks are resilient to credit defaults from Brazilian states, notably those under severe fiscal stress. Financial contagion remains small due to the high levels of capitalization that Brazilian banks, on average, maintain. State-owned banks are the most sensitive to credit defaults from Brazilian states, partly due to the fact that they are the largest creditor banks and because they take central positions in the network structure, thus making them more susceptible to financial contagion.

The major creditor of the Brazilian states is the federal government, and not the banking sector, except for the state of Goiás. However, it is still important to understand the resilience of the financial system to these external shocks, because it is able to largely amplify shocks. Therefore, small and localized shocks could develop into systemic events through the unsecured exposures between economic agents. Bank capitalization plays a major role as a financial shock barrier to hold back and control the level of amplification of financial shocks.

Other indirect channels of shock transmission to the financial system through public entities are relevant. For example, nonpayment to state employees would lead to an increase in their loan defaults to the financial sector. In addition, government indebtedness could force a reduction of investments, leading to increased unemployment and idleness of firms, and these, in turn, would have problems to honor their debts with the financial system. Finally, some of the bank credit that Brazilian states take are guaranteed by the federal government. Therefore, they are virtually risk-free if we assume that the federal government will not default. In this way, if we remove these guaranteed bank credit, then the exposure of the financial sector to fiscal risk would decrease.

References

Anand, K., Craig, B., and von Peter, G. (2015). Filling in the blanks: network structure and interbank contagion. *Quantitative Finance*, 15:625–636.

Aoyama, H. (2014). Systemic risk in Japanese credit network. In Abergel, F., Aoyama, H., Chakrabarti, B. K., Chakraborti, A., and Ghosh, A., editors, *Econophysics of Agent-Based Models*, New Economic Windows, pages 219–228. Springer International Publishing.

Bardoscia, M., Battiston, S., Caccioli, F., and Caldarelli, G. (2015). DebtRank: a microscopic foundation for shock propagation. *PLoS ONE*, 10(6):e0130406.

Battiston, S., Farmer, J. D., Flache, A., Garlaschelli, D., Haldane, A. G., Heesterbeek, H., Hommes, C., Jaeger, C., May, R., and Scheffer, M. (2016). Complexity theory and financial regulation. *Science*, 351(6275):818–819.

Blume, L., Easley, D., Kleinberg, J., Kleinberg, R., and Tardos, E. (2013). Network formation in the presence of contagious risk. *ACM Transactions on Economics and Computation*, 1(2):6:1–6:20.

Gai, P., Haldane, A., and Kapadia, S. (2011). Complexity, concentration and contagion. *Journal of Monetary Economics*, 58(5):453–470.

Glasserman, P. and Young, H. P. (2015). How likely is contagion in financial networks? *Journal of Banking and Finance*, 50:383–399.

Hojman, D. A. and Szeidl, A. (2008). Core and periphery in networks. *Journal of Economic Theory*, 139:295–309.

IMF (2015). Monetary policy and financial stability. *International Monetary Fund Policy Papers*.

IMF, BIS, and FSB (2009). Guidance to assess the systemic importance of financial institutions, markets and instruments. Report to the G20 Finance Ministers and Governors. *International Monetary Fund, Bank for International Settlements, and Financial Stability Board*.

KMV (2001). *Modeling Default Risk*. KMV corporation.

Lux, T. (2015). Emergence of a core-periphery structure in a simple dynamic model of the interbank market. *Journal of Economic Dynamics and Control*, 52(0):A11–A23.

Merton, R. (1974). On the pricing of corporate debt: The risk structure of interest rates. *The Journal of Finance*, 29(2):449–470.

Poledna, S., Molina-Borboa, J. L., Martínez-Jaramillo, S., van der Leij, M., and Thurner, S. (2015). The multi-layer network nature of systemic risk and its implications for the costs of financial crises. *Journal of Financial Stability*, 20:70–81.

Silva, T. C., Silva, M. A., and Tabak, B. M. (2017a). Systemic risk in financial systems: a feedback approach. *Journal of Economic Behavior and Organization*, 144:97–120.

Silva, T. C., Souza, S. R. S., and Tabak, B. M. (2016). Network structure analysis of the Brazilian interbank market. *Emerging Markets Review*, 26:130–152.

Silva, T. C., Souza, S. R. S., and Tabak, B. M. (2017b). Monitoring vulnerability and impact diffusion in financial networks. *Journal of Economic Dynamics and Control*, 76:109–135.

Souza, S. R. S., Silva, T. C., Tabak, B. M., and Guerra, S. M. (2016). Evaluating systemic risk using bank default probabilities in financial networks. *Journal of Economic Dynamics and Control*, 66:54–75.