Competing for Loan Informal Seniority: Theory and Evidence

Preliminary. Please do not circulate.

Abstract

Using data on the universe of credit card loans in Brazil, we show that universal default is rare: when borrowers are in distress, they often repay one of their credit cards. We then document that borrowers prioritize repaying the cards with higher limits, originated by fintech companies or by lenders that also sold them other financial products. Motivated by these facts, we introduce the concept of informal seniority and develop a model of non-exclusive lending to analyze its implications. We show that, due to competition for informal seniority, bankruptcy laws that impose the same recovery rate on all unsecured loans can decrease welfare.

Keywords: Debt dilution, Non-exclusive contracts, Selective default
JEL Classifications: G21, G51, D62

1 Introduction

Individuals frequently borrow from different financial institutions. Theoretically, the welfare implications of non-exclusive loan contracts—that is, the ability to borrow from multiple lenders—are ambiguous. On the one hand, non-exclusivity can benefit borrowers by letting them freely shop for the best products without being locked in by previous decisions. Additionally, it permits consumers to leverage potential synergies across various loan products and lending institutions. On the other hand, non-exclusive contracts can generate a debt dilution problem: lenders may impose a default externality on others as originating a new loan can affect borrowers' ability and incentive to pay off existing loans (e.g., Bizer and DeMarzo, 1992; Green and Liu, 2021).

The debt dilution problem potentially plays a more important role in the markets for unsecured household loans, where default rates are higher and recovery rates are typically smaller due to the absence of collateral. Moreover, since these markets lack of a formal seniority structure, lenders may seek to obtain informal seniority to increase the likelihood of repayment in the event of distress. In this paper, we use rich administrative data from Brazil to provide new evidence on multi-lender borrowing, debt dilution, and borrowers' repayment patterns in the credit card market. We then develop a model featuring non-exclusive contracts in which lenders compete to obtain repayment seniority.

Brazil's credit card market offers an ideal setup for studying debt dilution. First, the Central Bank of Brazil credit registry allows us to link each individual to their loan products and associated lenders, as well as loan-specific repayment decisions. Second, borrowing form multiple lenders is prevalent: out of 85 million credit card users in June 2022, 39.4 million (46.5%) had multiple active cards issued by two or more banks (Central Bank of Brazil, 2022). These users tend to borrow more than those with a single card, accounting for over 70% of the total outstanding credit card balance. Third, default rates are high: during our sample period, 9% of credit card borrowers were in default on at least one card.

We start by presenting new facts about the credit card market. First, we show that between 2019 and 2022, the share of credit card owners with multiple active cards issued by different financial institutions increased from 36% to 46%. Next, we focus on a sample of individuals who were in good standing with all their cards but missed a payment. Six months after missing a payment, the probability of being in arrears for 90 days (i.e., default) on at least one card is 46%, and it rises with the number of credit card lenders. Specifically, the probability is 43% for borrowers with one card lender, 45% for those with two card lenders, and 47% for those with three card lenders. Then we show that borrowers with multiple card lenders rarely default on all of them. Among those who default on at least one card, only 16% default on all of their cards, meaning that 84% continue to make payments on at least one card.

Selective defaulters are more likely to repay the cards with the highest credit limit, cards originated by fintech companies or cards issued by lenders that also sold them other products (e.g., auto loans). We show that banks continue to extend credit to selective defaulters as long as they are repaid. Limits and monthly usage levels drop to zero for the cards that selective defaulters do not pay. In contrast, limits and usage levels do not change for the cards they repay. This implies that default costs are primarily imposed by the bank that suffers the default.

The above results imply that debt dilution operates via two margins in the credit card market. First, the issuance of an additional credit card is positively correlated with the borrower's probability of default on any of their loans. This can be explained by riskier borrowers choosing to own multiple cards (adverse selection) and by the causal impact of having a credit card (moral hazard). Second, the issuance of an additional credit card may impact existing lenders if borrowers prioritize the new credit card over their existing loans. The results also suggest that banks strategically invest in securing informal loan seniority (i.e., ensuring they are the lenders borrowers prioritize for repayment), for example, through the use of contract terms, the cross-selling of additional financial product, and more effective collection practices.

Guided by these empirical facts, we develop a novel model to analyze and quantify the implication of competition for loan seniority. In the model, each lender simultaneously offers a loan contract, and borrowers can choose how many contracts to accept. For exposition purposes, we refer to the contract as a credit card contract.

The borrower can use multiple cards to leverage and thus consume more at the risk of being able to repay only a subset of cards. Formally, borrowers use credit cards to consume in the first period. In the second period, the borrowers choose whether to repay the lenders subject to an income constraint. They lose the card if they do not repay its balance and derive some utility from keeping each card in the second period. We consider that the utility in the second period is proportional to each card quality. In practice, a high-quality card can refer to a card offering substantial rewards, providing a fast-growing credit limit, or being linked to the promise of getting another financial product (e.g., a mortgage).

Lenders can choose the loan interest rate, and the credit limit, and can invest to increase the borrower's perceived value of repaying the card (i.e., the card quality). The key externality is that issuing a credit card reduces the borrower cost of defaulting on other cards. The reasoning is that defaulting on all card is more costly than on only a subset as the borrower needs at least one card to cater to his fundamental needs in the second period.[1] In line with what is observed in the data, lenders cannot undo this friction by designing loan contracts contingent on the terms of those signed with other lenders.

When issuing a loan, lenders internalize that the borrower may get another loan from a competitor and may be able to repay only a subset of them. As a result, lenders either overinvest or underinvest in the card quality relative to the exclusive contract case. Incentives to overinvest arise because of an arms race: offering a higher quality card relative to other banks increases the probability that the borrowers prioritize repaying this card. Incentives to underinvest come from lenders expecting borrowers

[1]We also consider that lenders cannot punish borrowers so much conditionally on default that they do not want to own more than one credit card. In practice, lenders can lower the borrower's credit score and cancel the credit card. Those may have a limited impact if the individual does not want to borrow from that bank in the future because he owns other cards. Credit card loans are recourse, but the legal costs are so high that, in practice, recourse is not used in this market.

to default more often under non-exclusivity and thus benefit less from the credit card quality.

We show that over or underinvestment in credit card quality can increase or decrease credit limits relative to the exclusive contract case. Indeed, the increase in default leads to a higher cost of lending and, thus, lower loan size. However, the potential underinvestment in card quality makes the cards cheaper to originate, which leads to lower interest rates and higher loan sizes.

The welfare impact of contract exclusivity is ambiguous. Debt dilution decreases welfare by generating too much default. However, the competition for loan seniority can increase welfare as it may reduce the markup on other financial products banks offer to gain seniority. The intuition for this result comes from interpreting the credit card quality as offering a personal loan (or any other type of loan) in the second period. In that case, the overinvestment in credit card quality can be interpreted as lowering the markup on personal credits relative to the exclusive credit card contract case. The welfare impact depends on the degree of competition in the personal loan market. When the personal credit market is perfectly competitive, competing for loan seniority forces to cross-subsidize products (i.e., making losses on personal loans and increasing credit card rates), which is not necessarily what would happen in the first best. Consequently, exclusivity (or seniority regulations) solves the debt dilution problem but can give lenders more market power in other markets.

The model also implies that, even if default and debt dilution are low in equilibrium, the threat of it existence is enough to induce large contract distortions to prevent it. For instance, let us consider a model in which the first best credit card contract would be offered absent debt dilution. To gain seniority, lenders must distort credit card terms away from the first best. When a lender has a comparative advantage in implementing this distortion, he gains seniority, mitigating the debt dilution problem.

We connect with several strands of the literature. First, we build on papers that study sequential and non-exclusive lending. The contributions in this field are mostly theoretical (e.g., Bizer and DeMarzo, 1992; Kahn and Mookherjee, 1998; Parlour and

Rajan, 2001; Bennardo et al., 2015; Attar et al., 2019; Green and Liu, 2021). We study a market – household credit card loans – where seniority cannot be contractually established.[2] Moreover, in this market, lenders cannot use collateral to exclude other lenders (e.g., Donaldson et al., 2020). Motivated by our empirical finding that default is rarely universal, we contribute to this literature by developing a model in which lenders can make investments to obtain informal loan seniority.

Empirical papers on non-exclusivity in lending markets include Degryse et al. (2016) and De Giorgi et al. (2023). We add to these papers by documenting the extent of selective default, its drivers, and implications for borrowers.[3] Furthermore, we develop a model that incorporates competition for loan seniority to analyze and quantify the cost of debt dilution on contract terms and welfare.

We also contribute to the literature on credit card borrowing.[4] Credit cards are typically the most common form of borrowing among households and are an important source of short-term financing for both consumers and businesses.[5] We show that credit risk is bank-specific and that banks compete for priority in this market due to selective default and non-exclusivity. We also study equilibrium consequences and policies through the lens of a model.

By showing that other loan products affect credit card repayment decisions, we also contribute to the literature on cross-selling (e.g., Robles-Garcia et al., 2022; Basten and Juelsrud, 2023). In our setting, we highlight a source of complementarity across products that emerges from a strengthening of the relationship with borrowers, which results in repayment prioritization. To the best of our knowledge, this source of economies of scope has not been analyzed empirically and theoretically.

Finally, we also contribute to the literature on fintech lending (e.g., Berg et al., 2022).

[2] Bizer and DeMarzo (1992) argue that debt dilution exists even with a formal seniority structure in the presence of moral hazard.

[3] Selective default has been documented in other contexts. Schiantarelli et al. (2020) show that firms default more on financially weaker banks as they perceive the value of the relationship with these banks as smaller. They also show that these effects are stronger when legal enforcement is weaker.

[4] See, for instance, Agarwal et al. (2015), Fulford (2015), Stango and Zinman (2016), Ponce et al. (2017), Keys and Wang (2019), Galenianos and Gavazza (2022), Nelson (2023), Berger et al. (2024), Castellanos et al., 2024, Fulford and Schuh (2024), and Matcham (2024).

[5] See Demirgüç-Kunt et al. (2022), Fonseca and Wang (2022), and Nanda and Phillips (2023).

We show that while the entry of fintech companies increases competition, it may impose externalities on previous lenders.

The rest of the paper is organized as follows. We first present the institutional setting and the data in Section 2. Section 3 shows the descriptive statistics, stylized facts, and reduced form regressions. Finally, Section 5 discusses the model.

2 Institutional Setting and Data

2.1 The Credit Card Industry in Brazil: Growth and the Extent of Multi-lender Borrowing

The credit card industry has expanded rapidly. After a stagnant period, the number of active credit cards doubled and the total amount of credit card loans grew by 73% between 2018 and 2023 (Appendix Figure A1). The growth in the number of cards was driven by two factors: the addition of new borrowers and the acquisition of cards from multiple banks by existing borrowers. According to Central Bank of Brazil (2022), the number of credit card users increased by 31% between June 2019 and June 2022, from 65 million to 85 million (40% of the population). While in June 2019 38% of users had active cards issued by two or more banks, in June 2022 this number was 47% (Appendix Figure A2). The share of credit card debt held by borrowers with more than one lender increased from 52% in 2018 to 72% in 2023.[6]

The expansion of digital institutions was a key contributor to the growth in the number of credit card users and active credit cards (Appendix Table A1). The number of credit card users with accounts in digital banks grew from 8.9 million in June 2019 to 36.5 million in June 2022 (310% growth). The credit card loans market has experienced a drop in concentration measures (Appendix Figure A3). The Herfindahl–Hirschman index (HHI) dropped from 0.17 in 2018 to 0.11 in 2023.

In Appendix Figure A4, we decompose the total credit card balance into three parts:

[6] In June 2022, 47% of credit card users had cards issued by two or more banks. These individuals accounted for 70% of the aggregate credit card debt, implying that they borrow more than those with cards issued by a single lender.

non-interest-bearing purchases, revolving debt, and buy now, pay later. Non-interest-bearing purchases refer to transactions made during the billing cycle that are fully due on the next due date and incur no interest charges if paid by that date; revolving debt refers to the accumulation of debt resulting from not paying the full balance of the monthly credit card bill; and buy now, pay later debt refers to card transactions where payment is split into installments, with or without interest charges. The composition of total credit card debt has remained stable, with non-interest-bearing purchases accounting 80% of total debt, revolving debt 10%, and buy now, pay later 10%. Despite no significant changes in composition, default rates have shown an upward trend since 2018. For revolving debt, the share of total balance in default exceeded 50% in 2023, while for buy now, pay later, this share reached 10%. Appendix Figure A5 shows that credit card revolving and buy now, pay later have the highest interest rates among the main loan types, with monthly interest rates reaching more than 10% for revolving debt.

Appendix Figure A6, we show that the participation of credit card debt in total household debt increased from 12.7% to 15% percent between 2018 and 2023. Credit cards are the most common loan contract, accounting for more than 50% of total household loan contracts. Although credit cards represented 12.7% of total household debt in 2018, they accounted for 37.5% of the household debt overdue by more than 15 days. This share increased to 55% in 2023.

2.2 Data and Sampling

We assemble data from different sources. We obtain monthly loan-level data from the credit registry of the Central Bank of Brazil (*Sistema de Informações de Créditos*, SCR). The dataset contains the universe of bank loans – mortgages, personal credit, car loans, credit card loans, payroll-deductible loans, and others – above 200 BRL (around 35 USD). For each month, we observe outstanding loan amounts, maturities, interest rates, defaults, and anonymized bank and borrower identifiers. For credit cards, we observe the monthly credit card usage, the account balance, the interest rate, and the credit limit.[7] If a borrower holds multiple cards with the same bank in a given month (e.g., two cards from different card networks), we observe aggregate amounts across all of their cards. We complement credit information with data on wages and employment from the Ministry of Labor (*Relação Anual de Informações Sociais*, RAIS). We observe the income, the employer, and some sociodemographic characteristics (age, sex, type of work).

Our sampling procedure is described in Appendix Figure A2. For a given month t (cohort), we take a 1% random sample of individuals who were in good standing with all their credit card loans in month $t-1$ (i.e., no card with payments overdue by more than 15 days) but became more than 15 days overdue on at least one card in month t. We refer to these individuals who are potentially transitioning into default (i.e., more than 90 days overdue) as *borrowers nearing distress*. To reduce the total sample size, we apply this procedure every three months, starting in June 2019 and ending in December 2022 – a total of 15 cohorts. For each individual in cohort t, we also collect credit information for the months $m \in \{t-12, t-11, ..., t-1, t, t+1, ..., t+11\}$.

For each cohort, we also take a 0.025% random sample of individuals who are not part of the *borrowers nearing distress* population. These individuals include those who were not in good standing with all their cards at $t-1$ and those who were in good standing with all their cards at $t-1$ and remained in good standing at month t. We refer to these individuals as *all other borrowers*. We apply inverse probability weighting to estimate statistics for the population of credit card users. The final data is at the cohort-individual-bank-month level.

In Appendix Table A3, we report the default and delinquency rates for both groups in the month preceding the cohort. For the *all other borrowers* group, 7% of individuals have at least one card with payments overdue by more than 90 days (default), and 10.3% of borrowers have at least one card with payments overdue by more than 15 days (delinquent). By design, both the delinquency and default rates for the *borrowers nearing distress* group are zero. In Appendix Table A4, we report a total of 524,533 ob-

[7] The accuracy of credit card limit data improved after May 2021. In certain specifications, we will limit the data to the period following this month.

servations at the cohort-by-individual level, with 252,844 observations belonging to the *borrower nearing distress* group. Consistent with the growth of the credit card industry, more recent cohorts are larger than the older ones.

3 Stylized Facts about Non-Exclusive Lending and Default

3.1 Non-exclusive Lending in the Credit Card Market

In Figure 4, we present the evolution over time of the number of active credit card lenders (those with a positive balance) per borrower, along with the proportion of borrowers holding active cards from two or more lenders. For both the *borrowers nearing distress* group and the population, the number of active card lenders per borrower increased between 2019 and 2022.[8] *Borrowers nearing distress* tend to have more cards than the average borrower. In the population, the share of borrowers with two or more card lenders increased from 36% in June 2019 to 46% in December 2022. In the subsample of *borrower nearing distress*, this share increased from 46% to 58%.[9]

In Table 1, we show how borrower characteristics are correlated with the number of active cards issued by different lenders. In the population, wages display an inverted U-shaped relationship with the number of card lenders, initially increasing with the number of active card lenders and then decreasing. The share of individuals with a formal job increases with the number of card lenders. Individuals with more card lenders have a higher debt service to income ratio, a higher average limit utilization across their cards, and tend to rely more on other loans types.

In Table 2, we report default rates in the month before the cohort by category of number of card lenders. The share of borrowers with at least one card in default in the *borrower nearing distress* subsample is zero across all categories. In the general popu-

[8] In Appendix Table A5, we show that the number of credit relationships not involving card loans also increased, along with the number of credit relationships involving digital banks.

[9] In Appendix Table A6, we show that in the *borrower nearing distress* subsample, 47% of individuals have one card lender, 27% have two, 14% have three, and 12% have more than three.

lation, this share increases monotonically with the number of card lenders. However, for other loan types, default rates do not follow this pattern. In the *borrowers nearing distress* group, borrowers with one card lender have a higher probability of defaulting on other loans compared to borrowers with more than one card lender.

3.2 Stylized Facts About Default in the Credit Card Market

In this section, we provide three new stylized facts about default behavior in the credit card market. First, we show that many borrowers default on only a subset of their cards. Second, we show that lenders do not cancel the credit limit on borrowers who defaulted on a competitor's card. Finally, we show that borrowers tend to prioritize repaying cards with better terms and cards issued by banks that offer them other financial products.

Fact 1: Many borrowers default on only a subset of their cards. To derive the first fact, we focus on the *borrowers nearing distress* group. These individuals were in good standing with all their cards in month $t-1$ and then miss a payment (overdue for more than 15 days) in time t. We investigate whether these individuals default on all or a subset of their loans six months later, defining a loan as in default if it is more than 90 days overdue.

In Table 3, we show that 46% of *borrower nearing distress* become more than 90 days overdue on at least one card and that this number increases monotonically with the number of card lenders. This group of individuals can be decomposed into two subgroups: *universal defaulters*, who default on all cards (24%), *selective defaulters*, who default on some cards and repay others (21%).[10]

These figures include individuals who have only one card and, therefore, cannot default selectively. When we restrict the analysis to individuals with two or more cards, we find that the vast majority of defaults are selective. For example, 45% of *borrower nearing distress* with two card lenders become more than 90 days overdue on at least one card. This figure can be broken down into 11% of universal defaulters and 34% of

[10]The two numbers add to 45% and not 46% due to rounding.

selective defaulters. That is, conditional on defaulting on at least one card, 76%=34/55 of borrowers repay at least one card. This proportion increases with the number of cards held by borrowers: 87% for borrowers with three card lenders, 92% for borrowers with four card lenders, 94% for borrowers with five card lenders, and 97% for borrowers with six or more card lenders. When we weight these proportions by the share of borrowers in each category, we find that, among borrowers with two or more credit card lenders who default on at least one card, only 16% default on all of their cards.

While the term selective default could apply to individuals who repay a subset of their credit card loans, they may default on the card for which they lack sufficient resources to repay, suggesting that the default decision is driven solely by liquidity constraints rather than a deliberate choice. In this case, selective defaulters would default on the cards that constitute the majority of their debt, implying that the distribution of the share of the total balance in default would be concentrated above the 50% threshold. However, as shown in Figure 6, the distribution does not follow this pattern. Specifically, there is a large group of selective defaulters who default on cards accounting for less than 50% of their balance, suggesting that they had sufficient resources to pay these cards but chose not to.

Motivated by this fact, we create a more conservative measure of selective default. We identify borrowers who default on the cards that account for less than 50% of their total credit card debt. In Table 3, we show that selective defaulters (21%) are comprised of two groups: those that default on less than 50% of their credit card balance (9% of borrowers), and those that default on more than 50% of their credit card balance (12% of borrowers). That is, *at least* 43%=9/21 of selective defaulters made a deliberate repayment choice. The proportion of these borrowers also increases with the number of cards.

In Table 4, we show that selective defaulters typically hold more cards, have a greater number of bank relationships, and a higher debt service to income ratio than universal defaulters and no-defaulters. While selective defaulters have higher incomes and credit card balances compared to universal defaulters, their income and balances are lower than those of non-defaulters. Regarding other loan types, selective defaulters tend to have more outstanding loans than universal defaulters.

In Figure 5, we show that all default measures are increasing over time, potentially due to the expansion in the number of credit cards. In Appendix Figure A7, we present the same figures for individuals with at least two card lenders. In this subsample, the incidence of selective default is significantly higher, as we exclude individuals with only one card, who can only default universally. The figures confirm that the incidence of selective default is increasing over time.

In Appendix Table A3, we note that selective default implies that the share of credit card contracts in default (7%) underestimates the share of borrowers with at least one card in default (9.2%). This discrepancy becomes more pronounced for borrowers with more than two active lenders, where 7% of contracts are in default, while 12.2% of borrowers have at least one card in default.

Fact 2: Lenders do not cancel the credit limit of borrowers who defaulted on a competitor's card. Figure 7 provides a visual representation of the second stylized fact. We show that defaults entail the loss of the defaulted card, as the limit available and monthly card usage drop to zero. However, select defaulters keep the limit and usage amounts of the cards they repay. In other words, as long as banks remain the prioritized lender, they continue to supply credit to selective defaulters.

Fact 3: Borrowers prioritize repaying cards with better terms. To understand the characteristics of the bank-borrower relationship that predict default (or repayment), we use the sample of selective defaulters that repay a subset of their loans and data at the borrower-cohort-bank level, that is, each observation is a bank-borrower credit card relationship. We estimate the following regression:

$$y_{icb} = \beta X_{icb} + \alpha_{ic} + \epsilon_{icb}$$

where y_{icb} is a binary variable that takes the value one if, six months after becoming overdue for more than 15 days in any card, individual i of cohort c defaults (i.e., is more than 90 days overdue) on the credit card issued by bank b. The vector X_{ibc}

contains borrower-bank characteristics measured in the month when the borrower becomes overdue for more than 15 days: limit available, limit utilization, global limit, loan balances, dummies for other active loan types, dummies for non-performance of other loan types, and dummies for bank types (digital or state-owned). We include borrower-cohort fixed effects (α_{ic}). We report descriptive statistics of the regression variables in Table 5. Selective defaulters default on 45% of their banks on average. In 69% of card relationships, borrowers used up all their credit card limits (100% of the limit); in 11%, more than 90% and less than 100%; in only 20% of card relationships, borrowers used less than 80% of the limit. 23% of relationships are with digital banks, 11% with state-owned banks, and the remainder (66%) with other banks. 6% of relationships contain an active payroll loan, 1% an active mortgage, 2% an auto loan, 12% a personal (consumer) loan, and 16% other loan types.

We report regression results in Table 6. Borrowers tend to default more on cards for which there is less limit available. In particular, default probabilities increase sharply (40%) on the card for which there is no limit available (100% utilization). Recalling that the mean of the dependent variable is 45%, the limit available seems to be an important driver of repayment choice.

Borrowers tend to default less on banks where they have a positive global limit, which includes limits for other loan types such as overdraft. Borrowers tend to default less on digital banks (-18%) and more on government banks (14%), meaning that borrowers may value their relationship with digital banks more than with other types of banks or that digital bank have a better debt recollection infrastructure.

In general, borrowers tend to default less on banks where they have other loan products: payroll (-6%), mortgage (-12%), auto loan (-5%), and other loans (-8%). However, default is more likely if the borrower has personal loans with the bank (4%). These results suggest that offering one loan type may increase the profitability of another, as it strengthens the bank's relationship with the borrower and improves its position in the repayment hierarchy. If the borrower already defaulted on other products from that bank, then it is also much more likely to default on the credit card from that bank rather than from another bank.

4 Difference-in-Differences

In Table 3, we show a positive relationship between the number of cards and default probabilities. This correlation may be due to higher-risk borrowers (e.g., those with more volatile incomes) being more likely to obtain additional cards. Alternatively, the correlation can be due to the causal impact of getting a card. The distinction between the two is crucial, as only the second channel results in debt dilution.

To determine the causal impact of obtaining a new card, we employ a difference-in-differences approach. We achieve this by leveraging certain institutional features of the Brazilian market. In Brazil, it is common for banks to establish partnerships with firms in order to secure the rights to process employee salary payments. The arrangement effectively mandates that employees receive their wages through the partner bank, increasing the likelihood that employees will obtain a card from that bank. We thus compare two groups of borrowers who worked at the same firm around the time the partner bank changed. The first group already had a relationship with the partner bank before it won (control), while the second did not (treatment). The identifying assumption is that the relative change in default probability between the two groups is due to the impact of the treatment group getting a new loan. Figure 1 summarises the research design.

We construct a worker-level, two-year panel centred around the month in which a new bank is assigned to manage a firm's payroll. We retain workers who remain employed at the same firm throughout the entire window, are active credit card users, and rely exclusively on this type of loan. We examine events (i.e., changes in banks managing payroll payments) occurring within the twelve months between May 2017 and April 2018.[11]

[11] We base our sample selection on two main considerations. First, in May 2019, a change occurred in the reporting standards for credit card loans, making April 2018 the last month for which we can observe an 11-month post-event window under a consistent reporting regime. Second, we focus on a period prior to the rise of digital banks as significant market players. Following their entry, the cost of

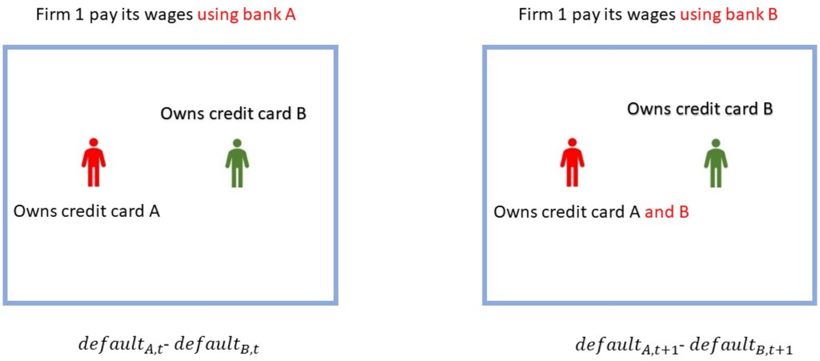

Figure 1: Identification strategy: we compare the relative default probability of the red and green groups of borrowers before and after the change in the bank that manages the payroll.

We estimate the following event-study regression to capture the dynamics of the effects:

$$y_{it} = \sum_{\tau=0}^{\tau=11} \beta_{-\tau} \times Treat_{i,-\tau} + \sum_{\tau=0}^{\tau=11} \beta_{\tau} \times Treat_{i,\tau} + \alpha_i + \alpha_{f(i)t} + \epsilon_{it} \quad (1)$$

where i indexes the individual, t the calendar month, and $f(i)$ the firm where individual i works. The variable $Treat_{i,t-\tau}$ is a dummy variable that takes the value one if individual i belongs to the treatment group and the new bank starts managing firm $f(i)$'s payroll τ months after t. The variable $Treat_{i,t+\tau}$ is a dummy variable that takes the value one if individual i belongs to the treatment group and the new bank starts managing firm $f(i)$'s payroll τ months before t. We control for time-invariant worker heterogeneity by including individual fixed effects, α_i, and we control for time-varying shocks to all workers of firm $f(i)$ by including firm-by-time fixed effects, $\alpha_{f(i)t}$. We cluster standard errors at the individual level.

We also estimate the following specification at the individual-month level:

$$y_{it} = \beta \times Post\_Treat_{i,t} + \alpha_i + \alpha_{f(i)t} + \epsilon_{it} \quad (2)$$

obtaining a new credit card declined substantially, reducing the likelihood that our treatment would remain effective.

where the variable $Post\_Treat_{i,t}$ is a dummy variable that takes the value one if individual i belongs to the treatment group and period t is after the time period when a new bank new bank starts managing firm $f(i)$'s payroll.

Figure A8 plots the β_t coefficients over time. We can see that 15% of the treated group gets a new card one year after the event. The take-up rate is not immediate. For instance, it takes 6 months for the take-up rate to reach 10%. The amount of card debt increases, indicating that the new card is used. We also observe that the number of defaults increases, primarily 6 months after the event. This is to be expected given the take-up rate and the fact that default is defined as being 3 months in arrears.

Table A7 reports the results of regression 2. We find that the treatment increases the number of cards and the delinquency.

5 Model

Section 3.2 provides suggestive evidence that lenders can use credit card terms (e.g., credit limit) and cross-sell other products to gain informal seniority. To study the impact of competing for informal seniority on contract terms and welfare, we develop a model with non-exclusive loan contracts.

In our model, the economy is populated with two groups of agents: borrowers and lenders. There is an arbitrary number of borrowers indexed by i and an arbitrary number of lenders indexed by c.

There are two periods in which agents make decisions. At the beginning of period 1, lenders simultaneously offer loan contracts, and borrowers choose how many contracts to accept. The set of card borrower i chooses is denoted $\mathcal{C}_i$. In the second period, borrowers choose the contracts they repay. The set of card borrower i repay is denoted $\mathcal{R}_i$.

For exposition purposes, we refer to a contract as a credit card. Each credit card c has a a credit limit L_c in period 1 and a gross interest rate r_c. We denote $R_c := L_c \cdot r_c$ the debt face value, due in period 2. The card also has characteristics that depend on the repayment behaviour, which we denote by the vector $q_c(\mathcal{R})$. To streamline the notation, we denote q_c instead of $q_c(\mathcal{R})$ when unambiguous, and refer to q_c as the card quality. In the main section, we interpret the vector q_c as a rewards system that provides the borrower with benefits at the end of the second period depending on the repayment behavior. We show in the appendix that our parametric assumptions are general enough so that q_c can be interpreted, in an extended three-period model, as the credit card terms (credit limit and interest rate) in the second and third periods, or as other products being offered in the second period (e.g., mortgage, or other loans).

The following subsections describe the borrowers (Section 5.1) and the lender's (Section 5.2) behaviour.

5.1 Borrowers

This section describes the borrower i choice of cards and repayment behaviour. We drop the i index in this section for clarity of the notation.

To build intuition, we begin by providing a micro-founded version of the borrower problem (Section 5.1.1). In Section 5.1.2, we then show how to parameterise the micro-founded model to get the functional form used in Section 5.1.2 and in the structural model. While the micro-foundation of Section 5.1.1 provides intuition, the reduced form approach of Section 5.1.2 can be micro-founded by linearising any model in which the utility function is increasing in the loan size and decreasing in the debt face value.

5.1.1 Microfoundation

The borrower i gets income W_t in each period ($t \in \{1, 2\}$). In period 1, the borrower can borrow using credit cards in order to consume. Their budget constraint in the first period is denoted $c_1 p_1 = \sum_{c \in \mathcal{C}} L_{c,1} + W_1$, where $c_1 p_1$ is the consumption basket time its price. In period 2, the borrower chooses how much to consume and whether to repay credit card debt. Their budget constraint is $c_2 p_2 = W_2 - \sum_{r \in \mathcal{R}} R_{r,2}$. The set of cards borrowers repay in period 2 ($\mathcal{R}$) must be feasible. That is, the amount the borrower repays ($\sum R_c$) cannot exceed their disposable income in the second period (W_{t+1}). We define the feasible set $F(\mathcal{C})$ as:

$$\mathcal{R} \subset F(\mathcal{C}) := \{f \subset \mathcal{C} : \sum_{c \in f} R_c \leq W_{t+1}\} \tag{3}$$

Guided by the data, we start by making two simplifying assumptions. First, we consider that borrowers max out the credit limits (L_c) on each card $c \in \mathcal{C}$ they own. This can be micro-founded with a model in which the marginal utility of consumption is higher in the first period than the marginal utility of consumption in the second period times the lender's marginal cost of lending. Second, conditional on repaying a card in period 2, the borrower repays it entirely. This behaviour can be micro-founded by lenders optimally choosing to cancel the borrower's card if it is not repaid fully, thereby maximising repayment probability (see Appendix A2). Those two assumptions simplify the borrower problem: they only choose the set of cards to own and repay, rather than the amounts they borrow and repay on each card they own.

The borrower's i utility in period 1 is the sum of their utility derived from consumption in periods 1 ($u(c_t, \mathcal{C})$) and 2 ($U(c_{t+1}, q, \mathcal{R})$), plus the indirect utility of owning a card at the end of period 2 (captured by q in $U(c_{t+1}, q, \mathcal{R})$). The borrower chooses consumption c_t, the set of cards to own $\mathcal{C}$, and the set to repay $\mathcal{R}$, to maximize their utility subject to their budget constraint:

$$\begin{aligned} u(L, R, q) = \max_{c_t, \mathcal{C} \in P(B)} u(c_t, \mathcal{C}) + \beta E[\max_{c_{t+1}, \mathcal{R} \in F(\mathcal{C})} U(c_{t+1}, q, \mathcal{R})] \\ s.t.\ c_t p_t = \sum_{c \in \mathcal{C}} L_{c,t} + W_t \\ c_{t+1} p_{t+1} = W_{t+1} - \sum_{r \in \mathcal{R}} R_{r,t+1} \end{aligned} \tag{4}$$

We use the notation $L := (L_c)_{c \in [\![1,|B|]\!]}$, $(R_c)_{c \in [\![1,|B|]\!]}$ and $(q_c)_{c \in [\![1,|B|]\!]}$ for the vectors of credit limits, interest rates, and card quality available in the market. B is the set of cards banks offer, $|B|$ the number of cards offered and $P(B)$ the power set of B.

The fact that the utility in period 1 depends on the set of cards $\mathcal{C}$ captures the non-monetary benefits of cards (e.g., convenience as a means of payment). The card quality

q and the repayment set $\mathcal{R}$ in the second period utility capture the non-monetary costs and benefits of defaulting or repaying the card.

The borrower problem 4 highlights that getting more cards allows the borrower to consume more in the first period, but this also increases repayment, and thus the probability of defaulting in the second period.

5.1.2 Parametrization

We now make some parametric assumptions to go from problem 4 to the utility function used in the theoretical analysis and in the empirical model. Using the budget constraint to replace c_t in the utility function, assuming that utilities are linear in consumption, normalising $p_{t+1} = 1$ and defining $p_{t+1}^{-1} = \alpha$, we get:

$$u(L, R, q) := \max_{\mathcal{C} \in P(B)} \overbrace{\sum_{c \in \mathcal{C}} \alpha L_c + \varepsilon_{ic}}^{period\ 1} + \beta E_\epsilon [\overbrace{U(L, R, q, \mathcal{C})}^{period\ 2}] \tag{5}$$

$$U(L, R, q, \mathcal{C}) := \max_{\mathcal{R} \in F(\mathcal{C})} W_i - \sum_{r \in \mathcal{R}} [R_r - \epsilon_{ir}] + V(q, \mathcal{R}) \tag{6}$$

The random variables ϵ_{ic} and ε_{ir} are borrower card-specific variables that affect the value of owning or not (and repaying or not) the card. Formally, ε_{ic} and ϵ_{ir} are drawn from continuous probability distribution functions. They are introduced to allow the demand and default functions, defined in the next section, to be continuous. ε_{ic} value is realized in period 1 and is observable by the borrower only. ϵ_{ir} is defined on a bounded set S. Its value is realized in period 2.

The function $V(q, \mathcal{R})$ represent the utility of repaying (when $c \in \mathcal{R}$) or not (when $c \notin \mathcal{R}$) cards. Specifying $V(q, \mathcal{R}) = \sum_c V_c(q_c, \mathcal{R})$, we can interpret $V_c(q_c, \{c\})$ as the utility derived from using the card c rewards at the end of period 2. $V_c(q_c, \{\emptyset\})$ captures the cost of not repaying card c. Examples include having a lower credit score conditional on not repaying the card or being unable to use the rewards because the lender canceled the card.

Appendix C discusses the micro-foundations for V_c across the three channels that allow lenders to gain seniority: by offering rewards, modifying the card terms conditional on repayments, or cross-selling products. The three channels are equivalent as they lead to a V_c function that is increasing in $q_c(\mathcal{R}) \in \mathbb{R}$.

5.2 Lenders

This section describes how lender c design their loan contract.

In period 1, lenders observe all parameters except the demand and repayment shocks (ϵ_{ic} and ε_{ir}). As lenders know the borrower's willingness to pay for the loan (i.e., $\frac{\alpha}{\beta}$), the equilibrium in which the lender chooses the pricing schedule $r(L)$ —i.e. the interest rate as a function of the amount borrowed— is equivalent to the one in which lenders directly choose the repayment $R := r(L)$ and the loan size L (i.e., the credit limit).

For each borrower i, lender c chooses the card loan size L, its repayment R, and quality q to maximise static profits conditional on their expectations about other lenders' offers and borrowers' behaviour. The profits is the probability of the borrower accepting the contract ($D_c(\cdot)$) times the net present value of the credit card contract ($NPV(\cdot)$). Investing in the card quality q_c costs $c(q_c, \theta_c(R_c, q_c))$ and increases the probability of the borrower repaying the card $\theta_c(R_c, q_c)$. Depending on the interpretation of q_c, the costs may be paid before knowing if the borrower repaid. Formally, we have:

$$\max_{\{L_c, R_c, q_c\}} D_c(u(R_c, L_c, q_c))[NPV(R_c, L_c, q_c, \theta(R_c, q_c))] \tag{7}$$

The demand function $D_c(u(R_c, L_c, q_c)) := Pr(c \in \mathcal{C})$ is the probability that the borrower gets a card from bank c. $\theta(R_c, q_c) := Pr(c \in \mathcal{R})$ is the probability that they repays bank c. The probabilities are derived from the borrower problem (5), conditional on knowing the distribution of ϵ_{ir} and ε_{ic} but not their realization. Making the default probability depend on the contract terms offered by lender c as well as all its competitors is the main departure from standard models of non-exclusivity.

We consider the expected net present value of lending NPV is a function of the loan present value $(\theta(R_c, q_c)R)$, the cost of originating the card $(mc(\theta)L)$ and the cost of quality $(c_c(q_c, \theta_c(R_c, q_c)))$. Formally, we have:

$$NPV(R_c, L_c, q_c, \theta(R_c, q_c)) := \overbrace{\tau L}^{Profits\ from\ transacting} + \overbrace{p^r[\theta(R, q_c)R_c - mc(\theta)L]}^{Profits\ from\ revolving} - c_c((q_c, \theta_c(R_c, q_c)) \tag{8}$$

The quantity τL is the net profits made from fees. With some exogenous probability p^r (equal to 1 in the micro-founded example of section 5.1.1, and captured by α and β in the reduced form specification 5.1.2), the borrower revolves. In that case, the lender earns in expectation $\theta(R, q_c)R_c$ and pays the additional costs $mc(\theta)L$. Appendix D discusses micro-foundations for the function c_c for the three channels that allow lenders to gain seniority: offering rewards, modifying the card terms conditional on repayments, or cross-selling products. Until the normative analysis (Section 6.2.3), these distinctions are not relevant.

We consider that the marginal cost of lending $mc(\theta)$ is a function of the is the average survival probabilities of borrower i $(\theta := \frac{1}{|\mathcal{C}_i|}\sum_{c\in\mathcal{C}_i}\theta_c)$. One interpretation of this modelling is that the marginal costs of lending increase in default probabilities through capital requirements, and that lenders do not internalize the impact of their behaviour on capital requirements. In our model, the key distortion is on contract terms q_c. Those distortions are not qualitatively affected by our assumption about $mc(\theta)$. However, this modelling of $mc(\theta)$ allows to capture the standard effect of non-exclusivity on L_c: it increases the real cost of lending and thus lowers the equilibrium loan size as in Parlour and Rajan (2001). An alternative is to make default costly for the lender so that it destroys resources instead of just relocating borrowers' wealth across banks as in Parlour and Rajan (2001) for instance. The fact that lenders do not internalise the impact of default on capital requirements is made to simplify the equations.

6 Equilibrium Analysis

In this section, we analyse the properties of the model using a stripped-down parameterization, to focus on building intuition about the economics at play. In particular, we consider that all borrowers revolve, lenders are symmetric and earn no transaction fees (i.e., lenders' demand and net present value functions have the same parameters with $\tau = 0$ and $p^r = 1$).

6.1 Exclusive Contract Equilibrium

To build intuition, let us first analyse the exclusive contract equilibrium. That is, the case in which borrowers can accept at most one contract.

6.1.1 Relevant Parameters' Value

To focus on a relevant equilibrium in which borrowers get a credit card, we restrict the values that the parameters can take. Formally, the set of parameters must be such that the following conditions are satisfied:

Conditions A1:

$$\alpha - \beta mc(\theta) > 0, \ \forall \theta \tag{9}$$

$$V_c(q, \{c\}) + \epsilon_{ir} - R > V_c(q, \emptyset), \ \forall R \leq W_i, \forall \epsilon_{ir} \in S \tag{10}$$

Equation 9 ensures that lending generates social surplus. It asserts that the marginal utility of loan (α) is larger than the marginal cost of lending.

Equation 10 describes the condition for borrowers to have incentives to repay their loans so that the market does not break down. The equation states that defaulting on all cards is costly: the benefit of repaying the loan $(V_c(q, \{c\}))$ minus its cost $(\epsilon_{ir} + R)$ are always higher than the utility of not repaying the loan $(V_c(q, \{\emptyset\}))$. The fact that the condition 10 holds $\forall \epsilon_{ir} \in S$ is made for simplicity of the exposition, so that defaults never happen in the exclusive contract case.[12]

With those assumptions, it is optimal for the lender to cancel the cards when the borrower does not repay.[13] Without loss of generality, we thus normalise $V_c(q, \emptyset)$ to 0, set the card quality in the default state ($q_c(\emptyset)$) to zero and focus on $q_c = q_c(\{c\})$.

6.1.2 Equilibrium contracts

The exclusive equilibrium contract (R^E, L^E, q^E) when lenders are homogeneous is:

$$\theta^E = 1 \tag{11}$$

$$R^E = W \tag{12}$$

$$L^E = \overbrace{\frac{W - c(q^E, \theta^E)}{mc(\theta^E)}}^{Lender\ break\ even\ condition} - \overbrace{\varepsilon^{-1}}^{inverse\ demand\ semi-elasticity} \tag{13}$$

$$q^E := \{q : \underbrace{\frac{\beta V'(q)}{c'(q, \theta^E)} = \frac{\alpha}{mc}}_{Equalized\ Marginal\ rate\ of\ transformation} \} \tag{14}$$

Proof: See Appendix A1.

Technical considerations: To get a closed form formula for the demand semi-elasticity at equilibrium, we assume that demand has a logit form (i.e., $D_b(u) := \frac{exp(\sigma u_b)}{\sum_l exp(\sigma u_l)}$ where u_x is the borrower utility when getting a card at bank x). In a symmetric equilibrium, the semi-elasticity with respect to loan size is thus $\frac{1}{\alpha\sigma(1-\frac{1}{|B|})}$. We consider that the inverse demand semi-elasticity ε^{-1} is smaller than $\frac{W-c(q^*,1)}{mc(1)}$ so that $L > 0$.

Equilibrium contracts: Because of assumption 10, defaulting is costly for the borrower. The contract is thus set so that borrowers never default (i.e, $\theta^E = 1$ in equation 11).

Equation 12 states that the debt face value is equal to the borrower's income. It

[12]For lending to happen, it is sufficient to have the condition hold for some ϵ_{ir}.

[13]Borrower will never default, and the cost increases in the card quality in the default state.

is optimal to let the borrower borrow as much as their income allows (i.e., $R = W$) because of the positive net present value of lending assumption (equation 9),

The equilibrium loan size in equation 13 features two elements. The first one is the loan size if lenders were to break even ($\frac{W-c(q^E)}{mc(1)}$). The second one is negative and equal to the interest rate markup ($\varepsilon^{-1} > 0$). The markup is the inverse demand semi-elasticity with respect to loan size. It captures the fact that lenders have some degree of market power when the demand elasticity is large and can thus charge above the marginal cost of lending ($\frac{\varepsilon^{-1}}{L}$ for each dollar lent).

The fact that repayment R is bounded by income W implies a trade-off between lending L and quality q. The trade-off is captured by equation 14. Absent this trade-off, the optimal quality would be that the marginal benefit of lending equates to its costs: $\beta V'(q) = c'(q, 1)$. Because the borrower resource is limited by W, the lender needs to select whether to offer a larger credit limit (L) or a larger quality (q). Thus, the optimal contract is such that the marginal rate of transformation of lending equals the investment one (equation 14). It implies that the optimal quality is such that $\frac{mc}{\alpha}\beta V'(q) = c'(q)$. The higher is the value of lending (i.e., $\frac{mc}{\alpha}$ low), the lower is the card quality.

6.2 Non-Exclusive Contract Equilibrium

Let us now analyse the non-exclusive contract equilibrium.

6.2.1 Relevant Parameters' Value

Depending on the parameter values, the market can either break down, be equivalent to the exclusive contract equilibrium, or feature borrowers owning multiple cards.

For instance, if the cost of defaulting is too high for borrowers, the non-exclusive equilibrium contracts and the exclusive ones are identical. The reason is the following. Consider a contract with the same characteristics as the exclusive contract (L^E, R^E, q^E) described in the previous section. The debt capacity of a borrower accepting this contract is maxed out. Thus, getting any additional contract would trigger default, which is too costly for the borrower.

To focus on the situation where the exclusive and non-exclusive cases differ, we add some curvature to the default cost function. In particular, we consider that defaulting costs are decreasing in the number of active cards the borrower owns, so that defaulting on all but one card is not too costly. The condition can be interpreted as borrowers needing at least one card to purchase the goods essential for their survival.[14] When this condition is satisfied, a borrower accepts more than one contract even if one of them has the same characteristics as the exclusive first-best contract. There can be two cases. First, the market breaks down as lenders may expect borrowers to default so often that the net present value of lending is negative. The condition $\alpha - \beta mc(\theta) > 0, \ \forall\theta$, in equation 9 holding for all θ ensures that this situation does not happen. Second, the market does not break down, and the borrower gets multiple cards.

Let us now formally state the condition for the equilibrium to feature multiple cards. For simplicity of the exposition, we begin by stating the condition in the case where there are only two lenders and then generalise. To make the notation unambiguous, we now denote $V_c(q_c, \mathcal{R}, \mathcal{C})$ instead of $V_c(q_c, \mathcal{R})$ to distinguish between defaulting from one card when having one or when having two. Formally, the parameters must be such that:

Conditions A2:

$$\underbrace{\alpha - mc(\theta)}_{marginal\ loan\ utility} + \underbrace{V_c(q_c, \{c\}, \{c, b\}) + V_b(0, \{c\}, \{c, b\})}_{defaulting\ on\ one\ card\ when\ having\ 2} > \underbrace{V_b(q_b^*, \{b\}, \{b\})}_{not\ defaulting\ on\ 1\ card} \tag{15}$$

To get the intuition behind the derivation of above equation 15, let us consider a borrower who has already accepted one card from lender b. The lender b offered the "exclusive" contract with a zero markup and quality q_b^E (i.e., the first-best in the exclusive contract case). The borrower now considers whether to get a new card from lender c. Appendix A2 shows that this is the key decision driving borrowers' number of cards in our model.

[14]The curvature could be put elsewhere: in the loan size or by adding some non-monetary benefits of owning multiple cards.

A new card is offered by a lender c and accepted by the borrower if it generates profits for the lender c, and utility for the borrower. This condition is satisfied if (i) lender c expects the borrower will not default on card c with a positive probability, and (ii) if a new card increases the borrower's utility when priced at marginal cost for any loan size dL. Condition (i) is satisfied thanks to the repayment shocks ε_{ir} in equation 6. Condition (ii) is satisfied if:

$$\max_{r=\{c,b\}} [\underbrace{\alpha L_b + W + V_b(\{r\},\{c,b\})}_{utility\ not\ repaying\ lender\ b} + \underbrace{\alpha dL - mc(\theta)dL + V_c(\{r\},\{c,b\})}_{utility\ new\ card}]$$
$$> \underbrace{\alpha L_b + W - R_b + V_b(\{b\},\{b\})}_{utility\ repaying\ card\ b\ and\ not\ getting\ new\ card} \quad (16)$$

Canceling out the terms that appear on both sides and this condition, and setting R_b to zero to maximize the right-hand side, we get the condition A2.

Appendix A2 shows that borrowers choose to repay only one card in equilibirum. Thus, although the above condition holds for the two-card case, it can be generalised to any number of lenders by adding V_e to both side. Where V_e is the cost of defaulting on the other cards. By having the cost of defaulting increase with the number of cards in default, this equation pins down the number of cards offered in equilibrium. For simplicity, we consider that all borrowers chose the same number of cards and that this number is known by lenders.

6.2.2 Equilibrium contracts

With the parameter values satisfying conditions A_1 and A_2, each borrower gets a card from each lender in equilibrium and repays only one card. Thus, when lenders are homogeneous, each card has a default probability of $1 - \theta^{NE}$, with $\theta^{NE} = \frac{1}{|B|}$. The equilibrium contract $(R_b^{NE}, L_b^{NE}, q_b^{NE})$ for each bank b is:

$$\theta^{NE} = \frac{1}{|B|} \tag{17}$$

$$R_b^{NE} = W \tag{18}$$

$$L_b^{NE} = \theta^{NE}\frac{W - c(q_b^{NE})}{mc(\theta^{NE})} - \varepsilon^{-1} \tag{19}$$

$$q_b^{NE} := \{q : \frac{\beta V'(q)}{c'(q)}[\overbrace{\theta^{NE\prime} W \frac{c'(q)}{\beta}\frac{\alpha}{mc}}^{over\ investment} + \overbrace{\theta^{NE}}^{under\ investment}] = \frac{\alpha}{mc}\} \tag{20}$$

Proof: See appendix A2.

Technical considerations: To get a closed-form solution for the demand semi-elasticity (ε^{-1}), We make the following assumption about the demand. We consider that there are B types of lenders indexed by x. Within each type x, there is an arbitrary number of lenders N that compete for the x^{th} credit card of the borrower. That way the semi-elasticity is (i.e., $D_b(u) := \frac{exp(\sigma u_b)}{\sum_{l=1}^{N} exp(\sigma u_l)}$), the equilibrium semi-elasticity is $\frac{1}{\alpha\sigma(1-\frac{1}{N})}$. I consider that $\varepsilon^{-1} < \frac{W - c(q_b^{NE})}{\theta^* mc(\theta^{NE})}$ so that $L_B^{NE} > 0$ to focus on the case in which there is borrowing.

Equilibrium contracts: Appendix A2 shows that it is optimal for the lender to cancel the card conditional on not repaying the loan (to maximize repayment probability), and to offer a card that maximises the borrower's debt capacity (as in the exclusive contract case). Given those contract features, borrowers default on all but one of the cards. The expected survival probability in a symmetric equilibrium (equation 17) is thus one over the number of cards: $\theta_b^{NE} = \frac{1}{|B|}$.

The higher default probability affects both credit limits (equation 19) and the level of investment (equation 20). Contrary to a model without endogenous default and card quality (e.g., Bizer and DeMarzo (1992) or Parlour and Rajan (2001)), the implication of non-exclusivity on contract terms is ambiguous: there can be over- or under-investment with respect to the exclusive contract case.

Equation 20 highlights the two channels impacting the investment decision. The difference with the optimal level of investment (i.e., equation 14) comes from the terms $[\theta^{NE\prime}W\frac{c'(q)}{\beta}\frac{\alpha}{mc} + \theta^{NE}] \neq 1$. For simplicity of the exposition, I assume that $\beta^{-1} = \frac{mc}{\alpha}$ in equation 20 when discussing the different channels at play below. When doing the comparative static with respect to the exclusive contract case, this shuts down the fact that when the marginal cost goes up, the relative cost of lending versus investing goes up, so lenders substitute lending with investment and make our results independent of this assumption.

The terms $\theta' W$ is potentially larger than one. It can lead to over-investment due to an arms race channel: lenders have incentives to increase q to gain seniority, yet in equilibrium, other lenders also invest. In practice, this investment can represent too many rewards offered in the second period or the promise to make a loan below the break-even rate in the future. Interpreting the model as secured lending instead of credit cards, the overinvestment could represent asking for too much collateral to gain seniority.

The second term θ is smaller than one and leads to underinvestment. The reason is that borrowers are less likely to benefit from the continuation value of the card as they are likely to default, so lenders have fewer incentives to invest in q.

Equation 19 shows the effect of non-exclusive contracts, and thus higher default on credit limits. As borrowers get a number of cards equal to the number of lenders, I calculate the borrower i total amount of lending. This allows us to compare it with the exclusive contract case:

$$L^{NE} := \sum_{b=1}^{B} L_b^{NE} = \frac{W - c(q_b^{NE}, \theta^{NE})}{mc(\theta^{NE})} - B\varepsilon^{-1} \tag{21}$$

Equation (21) highlights the three channels that are affecting the loan size relative to the exclusive contract case.

First, with exclusive contracts, the cost of lending increases $mc(\theta^{NE})$. This effect tends to increase the break-even rate and thus decrease the equilibrium loan size. This is the first source of inefficiency: because the default is not too costly for borrowers when they own multiple cards, there is debt dilution in equilibrium, leading to too much default.[15] This is the standard effect of contract non-exclusivity highlighted in, for instance, Parlour and Rajan (2001), Bizer and DeMarzo, 1992 or Green and Liu, 2021.

Second, exclusive contracts distort incentives to invest in q. When there is over-investment, the cost of lending increases (i.e., $c(q^{NE}) > 0$ is higher than in the non-exclusive case), pushing the credit limit down. When there is underinvestment, the opposite happens. These results imply that looking only at the impact on credit limits to draw conclusions about welfare can be misleading.

Third, lenders set a markup ε^{-1} for each additional card, lowering the loan size of each card. This effect adds up to the number of cards. Imperfect competition thus becomes even more costly under non-exclusive contracts.

6.2.3 Welfare Analysis

This section analyses how competition for loan seniority affects welfare and whether personal bankruptcy procedures are desirable. We focus on the non-exclusive equilibrium, and study whether it can be improved by policies that impose all loans of the same formal seniority to have the same recovery rate.

For the interested reader, we analyse in the Appendix G when the exclusive contract equilibrium is more desirable than the non-exclusive one. The key trade-off is the following. Non-exclusive contracts offer borrowers a higher product variety and the possibility to mix and match products, however, it comes at the cost of a distortion in contract terms (over- or underinvestment). There is an additional benefit to offering non-exclusive contracts when gaining seniority involves offering another loan: competition for loan seniority provides incentives to lower the markup on the product.

The random shock modelling we use is convenient for its tractability, to bring the model to the data, and to study competition (see, for instance, Rochet and Stole (2002) for a theoretical discussion). Yet, it requires caution when making welfare statements.

[15] The assumption that $\alpha > mc(\theta^{NE})$ ensures that lending is still positive NPV. This allows the equilibrium to exist for an arbitrary number of lenders.

The reason is that random shocks affect both the default sensitivities to contract terms and the utility levels. To illustrate those two dimensions, let us consider the following parametric assumption. The repayment shocks ϵ_{ir} have the form σe_r with e_r following an extreme value distribution and σ parametrising its variance. With this parametric assumption the borrower survival sensitivity ($\frac{\partial \theta^{NE}}{\partial q^{NE}}$) and utility ($u_i^{NE}$) are:

$$\frac{\partial \theta^{NE}}{\partial q^{NE}} = \sigma\theta^{NE}(1-\theta)V'(q^{NE}) \tag{22}$$

$$u_i^{NE} := \sum_{c\in B} \alpha L_c^{NE} + \beta ln(\sum_{c\in B} exp\{\sigma V_c(q^{NE})\}) \tag{23}$$

$$= \alpha L_c^{NE} \cdot |B| + \beta[\sigma V_c(q^{NE}) + ln(|B|)] \text{ in a symmetric equilibrium} \tag{24}$$

The above equations summarise the two channels captured by the default shocks. Equation 22 shows that the parameter σ drives the default elasticity to changes in card quality. It is thus a key parameter that affects the competition for the loan seniority channel. However, as the shocks ϵ_{ir} enter the utility function, they also capture the borrower's love for product variety (similarly to the demand elasticity parameter in CES demand). This is why σ multiplies V in equation 24 and the utility increases with the number of products $|B|$ offered. To see that, one can compare the equation 24 utility (u_i^{NE}) to the utility of a borrower for which the default shock enters the default decision, but not the utility (i.e. $\tilde{u}_i^{NE} = \sum_{c\in|B|} \alpha L_c^{NE} + Pr(r \in \mathcal{R})V_c(q_c^{NE}) = \alpha L_c^{NE} \cdot |B| + \beta V_c(q^{NE})$).

To shut down the love for variety channel so that we can analyze the effect of changing the competition for loan seniority channel using changes in σ, we set $\beta = \frac{1}{\sigma}$ and consider that the utility is $\tilde{u}_i^{NE} := u_i^{NE} - \frac{\ln(|B|)}{\sigma}$. Scaling by $\frac{ln(N)}{\sigma}$ does not affect the contract terms, but it makes the utility $\tilde{u}_i$ is exactly equal to the utility without the error term affecting its value.[16]

To analyse the effect of the competition for the loan seniority channel, let us take

[16] By still allowing the error term to enter the utility function, we ensure that borrowers' decisions are optimal so that the loss in welfare does not come from mistakes in default choices.

the derivative of $\tilde{u}_i^{NE}$ with respect to σ:

$$\frac{\partial \tilde{u}_i^{NE}}{\partial \sigma} = [\overbrace{-\frac{c_1(q^{NE}, \theta^{NE})}{mc(\theta^{NE})} \cdot |B|}^{\textit{Lower loan size}} \overbrace{+V'(q^{NE})}^{\textit{Higher rewards}}] \cdot \frac{\partial q^{NE}}{\partial \sigma} \tag{25}$$

$\frac{\partial q^{NE}}{\partial \sigma}$ is positive, so the sign depends on $\mathcal{W} := [-\frac{c_1(q^{NE}, \theta^{NE})}{mc(\theta^{NE})} \cdot |B| + V'(q^{NE})]$. The equation captures the following trade-off: increasing the default sensitivities incentivises lenders to overinvest; however, this makes credit cards more expensive, which lowers the credit limit. One can estimate the c and V functions using standard industrial organisation approaches, such as Berry et al. (1995), and plug the estimates into $\mathcal{W}$.

To gain more intuition about the impact of competition on welfare, let us consider that $V(q) = \gamma q$ and $c(q) = c\frac{q^2}{2}$. In that case, we have:

$$q^{NE} = \theta^{NE}\gamma \frac{\sigma(1-\theta^{NE})W + \frac{mc}{\alpha}}{c} \tag{26}$$

$$\mathcal{W}(\sigma) = \gamma - q^{NE}[\frac{c}{mc}] \tag{27}$$

The equation 27 implies a unique competition for loan seniority level $\varepsilon : \frac{\partial \theta^{NE}}{\partial q^{NE}} \frac{1}{\theta^{NE}} = \sigma(1 - \frac{1}{|B|})$ that maximizes welfare. It is achieved for $\mathcal{W}(\sigma^{SB}) = 0$ and is equal to:[17]

$$\varepsilon^{SB} := [1 - \frac{\theta^{NE}}{\alpha}] \frac{mc}{\theta^{NE} W} \tag{28}$$

To understand equation 28, notice that, in our model, there can be underinvestment due to default being too large, and over-investment due to competition for loan seniority. The above equation imply that, when the overinvestment channel dominates (i.e., the default elasticity in the data is higher than ε^{SB}), policies that lower the competition for loan seniority increase welfare. One candidate policy is to force borrowers to repay each lender $\frac{W}{|B|}$ in the second period. This can be interpreted as

[17] Indeed: $\mathcal{W}(\sigma)$ is decreasing in σ. When $\sigma = 0$, $\mathcal{W}(\sigma) := \gamma[1 - \frac{\theta^{NE}}{\alpha}] > 0$ when $\alpha > \theta^{NE}$ and $\lim_{\sigma \to \infty} \mathcal{W}(\sigma) = -\infty$.

a default on any card triggering a bankruptcy procedure similar to the one for firms. The following statement and figures 2 and 3 summarize when such a policy is likely to be desirable.

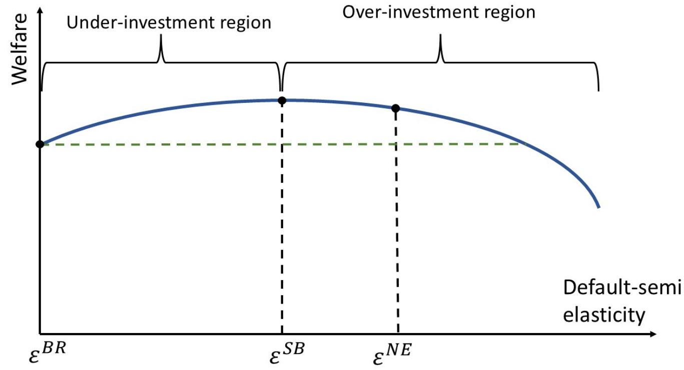

Figure 2: $(\varepsilon^{SB}, \varepsilon^{NE}, \varepsilon^{BR})$ **are the optimal, current and under regulation semi-elasticities.**

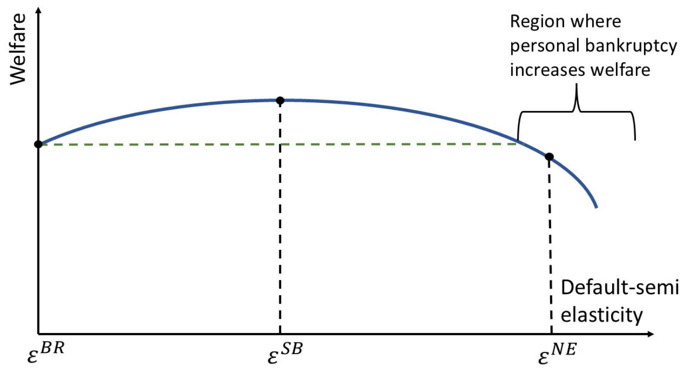

Figure 3: Example where the bankruptcy regulation is welfare increasing.

Result: *Setting a bankruptcy regulation that forces the same recovery rate (i.e., borrowers to repay all lenders* $\frac{W}{|B|}$ *instead of letting borrowers choose which lender to repay first) is valuable when competition for loan seniority is too high. This is more likely to be the case when the marginal costs of lending* (mc)*, default* $(1\text{-}\theta)$*, and the number of lenders are low* $|B|$*, or when borrowers' debt capacity is large* W *or value consumption in the first period is high.*

The above statement describes conditions that make competition for loan seniority detrimental to welfare, as it overcompensates the under-investment channel (as in Figure 3). The intuition for how it depends on the model parameters is the following (i.e., how the blue curve in Figure 3 changes with the model parameters). When the debt capacity is large (W), it creates incentives to compete for loan seniority more aggressively because there is more money at stake. This makes it more likely that ε^{NE} is in the right region in figure 3. When the number of borrowers and default are low, the underinvestment channel is likely to be small. As a result, the over-investment channel is more likely to dominate. Finally, as illustrated in equation 25, competing for loan seniority increases rewards q at the expense of the credit limit. This tradeoff is more likely to be detrimental for borrowers when borrowing is valuable (high α) or when borrowing is cheap (low mc).

7 Quantitative Analysis

7.1 Parametrization

We use the quantitative model to assess the cost of the debt dilution. We do so by solving for a counterfactual world in which contracts are exclusive. In the quantitative model, we abstract from multiple products being offered to focus on competition for seniority using card terms and monitoring.

The following parametric assumptions are such that the exclusive contract equilibrium can be solved using a contraction mapping.

Parametrization of lender's profit: For each observationally equivalent borrower i, the lender c problem is:

$$\max_{X,\xi} \overbrace{D_{ic}(X,\xi)}^{\text{Demand}} \left[\overbrace{\tau_{ic}}^{\substack{\text{Net profits}\\ \text{per transaction}}} L + \underbrace{\overbrace{p_r}^{\substack{\text{Revolving}\\ \text{probability}}} \left(\overbrace{\theta_{ic}(X)}^{\substack{\text{Survival}\\ \text{probability}}} r - mc \right)}_{\substack{\text{Expected profits}\\ \text{on revolving loans}}} L - \underbrace{mc^m(\xi)}_{\text{Marketing costs}} \right]$$

The variable $X = (r, L)$ represents the card interest rate r and the credit limit L, which are functions of the number of cards the borrower chooses ($\mathcal{C}_i$). The revolving probability p_r is estimated using revolving data; we consider that it does not depend on contract terms. The demand and survival functions (D and θ) are specified below and estimated using choice and default data. The net profit per transaction (τ) and the marginal cost functions (mc, mc^m) are inferred using the model's first-order conditions together with estimates of the demand and default elasticities.

Parametrisation of utilities: We consider that utilities are linear in the aggregate loan contract and interest rate. We also add a card fixed effect to capture the utility coming from the card.

$$\textbf{Demand: } U_i(\mathcal{C}_i) = \overbrace{\sum_{c \in \mathcal{C}_i} [\beta_1 L_c - \beta_2 r_c + \xi_c]}^{u_i(\mathcal{C}_i)} \cdot exp(\varepsilon_{\mathcal{C}_i}) \tag{29}$$

$$\Longrightarrow Pr(c \in \mathcal{C}_i) = \frac{\sum_{\mathcal{CS}_c \in P_c(B_i)} u_i(\mathcal{CS}_c)}{\sum_{\mathcal{CS}_c \in P(B_i)} u_i(\mathcal{CS}_c)},\ D_{ic}(X, \xi) := Pr(c \in \mathcal{C}_i | \mathcal{C}_i \backslash c) \tag{30}$$

$$\textbf{Default: } U_i^r(\mathcal{R}_i) = \overbrace{\sum_{c \in \mathcal{R}_i} [\beta_1^r L_c - \beta_2^r r_c + \xi_c^r]}^{u_i^r(\mathcal{C}_i)} \cdot exp(\varepsilon_{\mathcal{C}_i}) \tag{31}$$

$$\Longrightarrow \theta_{ic}(X) := Pr(c \in \mathcal{R}_i | \mathcal{C}_i) = \frac{\sum_{\mathcal{CS}_c \in P_c(\mathcal{C}_i)} u_i^r(\mathcal{CS}_c)}{\sum_{\mathcal{CS}_c \in P(\mathcal{C}_i)} u_i^r(\mathcal{CS}_c)} \tag{32}$$

We denote $P(B_i)$ the power set of all the banks available to the borrower (denoted B_i), and $P_c(B_i)$ the subset of $P(B)$ in which each element contains a card from bank c. We define similarly $P(\mathcal{C}_i)$ and $P_c(\mathcal{C}_i)$.

8 Conclusion

Access to credit has been growing, particularly in emerging economies, with credit cards playing a key role in this expansion. In this paper, we use a rich administrative dataset of household credit in Brazil to document new facts about the credit card market. We show that it is common for borrowers to hold multiple active cards issued by different lenders. We find that default rates rise with the number of lenders, and that, when facing financial distress, borrowers tend to prioritize repaying certain banks over others. Furthermore, we show that prioritized banks keep supplying credit to individuals even after they default on other banks.

While borrowing from multiple lenders may reflect increased competition and greater access to differentiated loan products and services, it also presents challenges, as externalities between lenders can lead to inefficiencies. Banks may respond to it by competing for repayment priority. We study these market inefficiencies through the lens of a model and, in future work, we plan to discuss policies that could mitigate them.

Although our focus is on the credit card market, the idea of lenders endogenously competing for seniority applies more broadly to contexts in which claimants compete for scarce resources that are not allocated according to rules set by regulation (e.g., bankruptcy law) or contractual agreements. In many countries, the lack of formal household debt resolution mechanisms means that providers of unsecured loan products, beyond just credit cards, also compete for repayment priority. Similarly, before filing for bankruptcy, both firms and individuals may strategically select which creditors to prioritize. Consequently, our framework provides valuable insights into the implications of non-exclusivity in a variety of other contexts.

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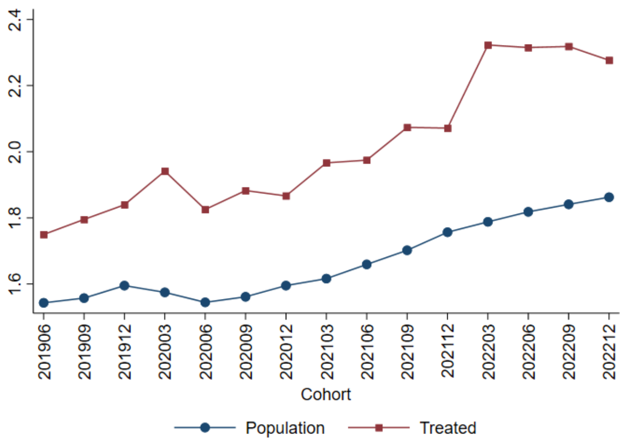
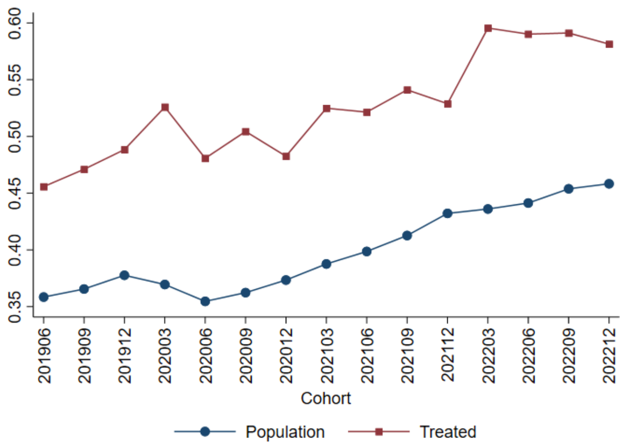
Figures

Figure 4: Evolution of borrowing from multiple card lenders



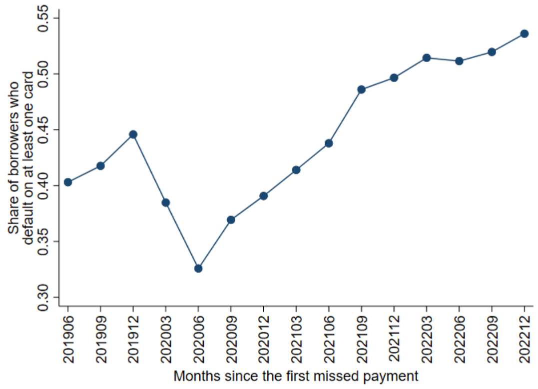
(**a**) Average number of credit card lenders



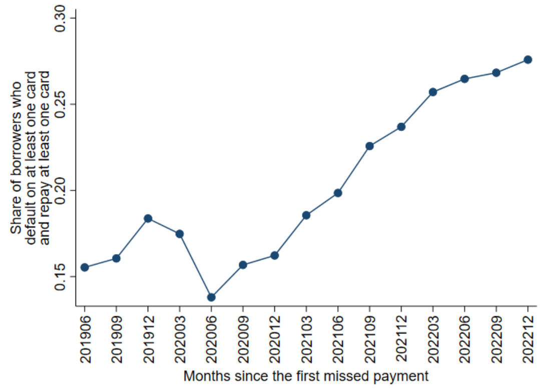
(**b**) Share of card borrowers with two or more credit card lenders

Notes: Data at the borrower-cohort level. We compute population averages using inverse probability weighting. The group *treated* refers to *borrowers nearing distress*.

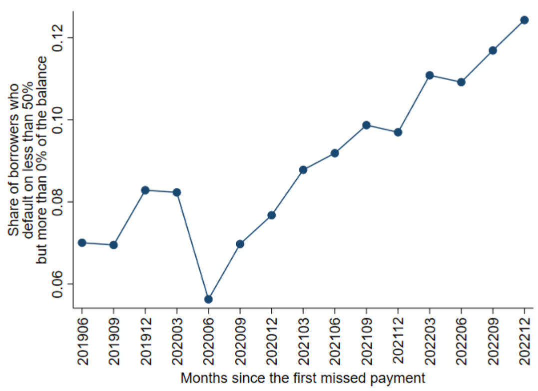
Figure 5: Default and selective default evolution



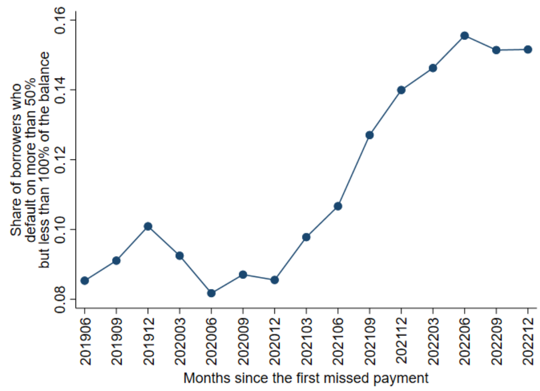
(**a**) Share of *borrowers nearing distress* with at least one card 90+ days overdue



(**b**) Share of *borrowers nearing distress* with at least one card 90+ days overdue and at least one in good standing



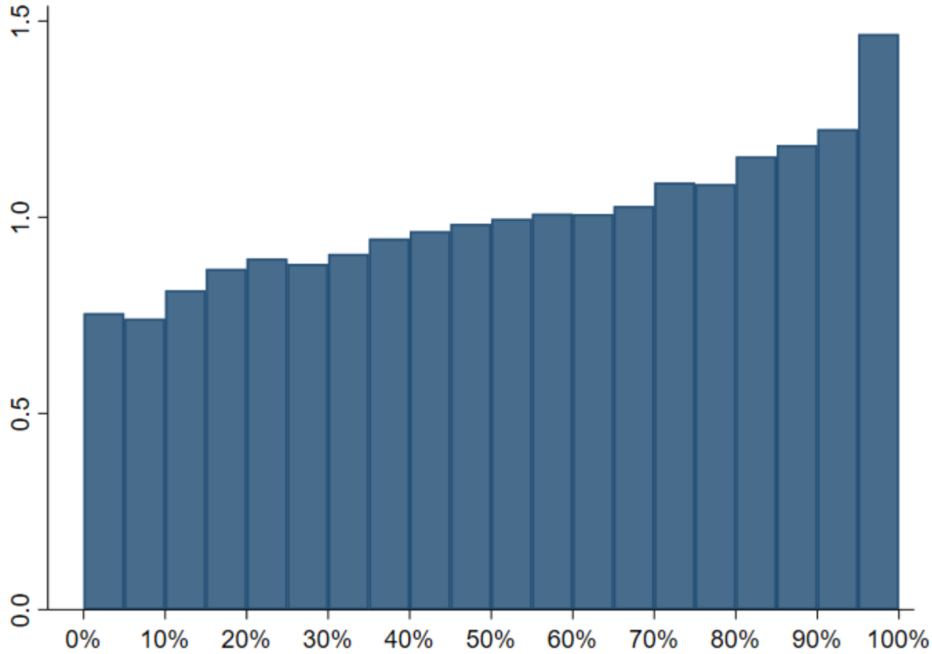
(**c**) Share of *borrowers nearing distress* with at least one card 90+ days overdue, at least one in good standing, and default on less than 50% of the total card balance



(**d**) Percentage of *borrowers nearing distress* with at least one card more than 90 days overdue and at least one card in good standing, and default on more than 50% of the total card balance

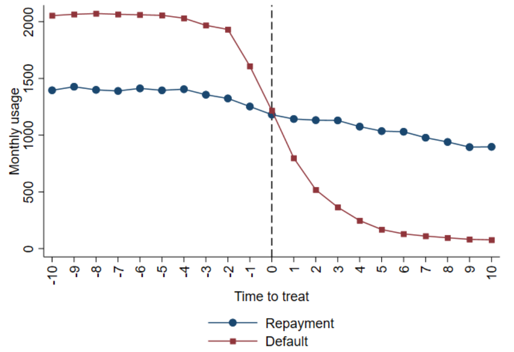
Notes: The sample is comprised of individuals in the *borrower nearing distress* group who have one or more cards. We classify a card as in default if there is balance overdue for more than 90 days six months after the borrower becomes more than 30 days overdue.

Figure 6: Share of balance in default

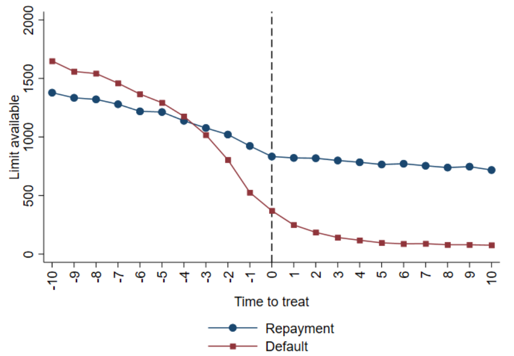


Notes: This figure plots the distribution of the share of total balance in default among selective defaulters in the *borrower nearing distress* subsample. We measure default as payment more than 90 days overdue six months after the cohort month. We compute the shares using the balances in the cohort month.

Figure 7: Evolution of the limit and card usage, cards repaid versus not repaid of selective defaulters



(**a**) Card monthly usage, card pair versus not repaid of selective defaulters



(**b**) Limit available, card pair versus not repaid of selective defaulters

Notes: The sample is comprised of individuals in the *borrower nearing distress* group who repay at least one card and default on at least one. Month 0 is the month when borrowers first miss a payment.

Tables

Table 1: Mean of selected variables by number of credit card lenders

	Number of card lenders					
	1	2	3	4	5	6+
Panel A: Random sample (population)						
Wage (formal)	3,757	4,124	4,171	3,812	3,841	3,920
Wage (reported)	3,214	3,704	3,766	3,607	3,737	3,663
Debt service to income	0.18	0.27	0.38	0.48	0.59	0.85
Formal sector	0.36	0.43	0.46	0.46	0.47	0.47
Age	45	44	44	44	43	41
Total balance card	2,631	5,386	7,985	10,300	13,335	20,069
Average limit utilization (%)	54	55	58	61	64	69
Share cards 100% utilization	0.22	0.22	0.24	0.26	0.29	0.34
Total card limit available	5,129	9,247	11,206	11,793	12,688	13,086
Dummy auto loan	0.07	0.12	0.16	0.20	0.24	0.26
Dummy mortgage	0.06	0.09	0.10	0.11	0.11	0.11
Dummy other loans	0.27	0.35	0.43	0.48	0.53	0.60
Dummy payroll loan	0.17	0.21	0.23	0.26	0.26	0.25
Dummy personal loan	0.14	0.20	0.27	0.34	0.42	0.53
Panel B: Borrowers nearing distress						
Wage (formal)	2,151	2,943	3,320	3,399	3,548	3,396
Wage (reported)	2,121	2,778	3,054	3,159	3,272	3,374
Debt service ratio	0.26	0.41	0.54	0.65	0.77	1.00
Formal sector	0.29	0.38	0.42	0.44	0.44	0.45
Age	39	42	42	42	42	41
Total balance card	1,558	4,234	6,950	9,702	12,621	19,724
Average limit utilization (%)	83	77	75	76	76	78
Share cards 100% utilization	0.74	0.53	0.47	0.45	0.44	0.45
Total card limit available	548	2,431	3,901	4,639	5,233	6,271
Dummy auto loan	0.06	0.11	0.16	0.20	0.22	0.24
Dummy mortgage	0.03	0.06	0.08	0.09	0.10	0.11
Dummy other loans	0.35	0.45	0.52	0.57	0.63	0.69
Dummy payroll loan	0.12	0.22	0.25	0.26	0.28	0.26
Dummy personal loan	0.21	0.30	0.36	0.43	0.48	0.59

Notes: Data at the borrower-cohort level. In Panel A, we compute population averages using inverse probability weighting. In Panel B, we compute sample means for the *borrower nearing distress* group. Wage (formal) refers to wages recorded in administrative data (RAIS) and are only available for workers employed in formal jobs. Wage (reported) refers to wages reported by banks. Formal sector is a dummy variable that takes the value 1 if the individual has a formal job. Debt service to income is the ratio of debt payments to income. All monetary amounts are in Brazilian reais (BRL).

Table 2: Mean of default variables by number of credit card lenders

	Number of card lenders					
	1	2	3	4	5	6+
Panel A: Random sample						
Default payroll loan	0.04	0.04	0.03	0.03	0.03	0.02
Default mortagage	0.01	0.01	0.01	0.01	0.01	0.02
Default auto loan	0.04	0.03	0.03	0.02	0.03	0.04
Default personal loan	0.11	0.10	0.10	0.10	0.11	0.13
Default other loans	0.10	0.08	0.09	0.09	0.09	0.11
Default credit card	0.07	0.10	0.13	0.15	0.17	0.21
Panel B: Borrowers nearing distress						
Default payroll loan	0.07	0.04	0.03	0.03	0.02	0.03
Default mortagage	0.05	0.02	0.01	0.01	0.00	0.01
Default auto loan	0.09	0.05	0.03	0.03	0.02	0.02
Default personal loan	0.09	0.08	0.06	0.05	0.05	0.05
Default other loans	0.11	0.07	0.05	0.04	0.04	0.07
Default credit card	0.00	0.00	0.00	0.00	0.00	0.00

Notes: Data at the borrower-cohort level in the month before the cohort. In Panel A, we compute population averages using inverse probability weighting. In Panel B, we compute sample means for the *borrower nearing distress* group.

Table 3: Default rates by number of cards, *borrower nearing distress* sample

Number of card lenders	Shares					
	Repay all cards	Default on at least one card	Default on all cards	Selective: Borrower defaults on card with		
				Largest balance	Smallest balance	Total
1	0.57	0.43	0.43	-	-	-
2	0.55	0.45	0.11	0.20	0.13	0.34
3	0.53	0.47	0.05	0.22	0.19	0.41
4	0.51	0.49	0.04	0.23	0.22	0.45
5	0.46	0.54	0.03	0.25	0.26	0.51
6+	0.36	0.64	0.02	0.35	0.27	0.62
Total	0.54	0.46	0.24	0.12	0.09	0.21

Notes: Data at the borrower-cohort level. Selective means that the borrower defaults on at least one card while repaying at least one other card. Selective largest balance means that borrowers defaulted on the cards that accounted for more than 50% of their total card balance measured in the month when they missed a payment. Selective smallest balance means that borrowers defaulted on the cards that accounted for less than 50% of their total card balance measured in the month when they missed a payment.

Table 4: Mean of selected variables by default category

	No	Selective		Universal
	default	Largest balance	Smallest balance	default
Number of banks	4.1	4.6	4.8	3.6
Number of card lenders	2.9	3.2	3.3	2.4
Number of digital banks	0.6	1.0	0.9	0.7
Share card balance digital banks	0.15	0.15	0.17	0.16
Number of state-owned banks	0.6	0.7	0.7	0.6
Wage (formal)	3,559	2,735	3,083	2,245
Wage (reported)	3,203	2,719	2,855	2,365
Debt service to income	0.46	0.68	0.56	0.48
Formal job	0.42	0.40	0.41	0.29
Age	43	39	42	36
Dummy payroll loan	0.25	0.20	0.29	0.12
Dummy mortgage	0.08	0.07	0.08	0.05
Dummy auto loan	0.15	0.15	0.14	0.10
Dummy personal loan	0.31	0.42	0.41	0.36
Dummy other loan types	0.50	0.55	0.53	0.46
Dummy default payroll loan	0.01	0.01	0.01	0.01
Dummy default mortgage	0.00	0.00	0.00	0.00
Dummy default auto loan	0.00	0.01	0.01	0.01
Dummy personal loan default	0.02	0.03	0.04	0.03
Dummy other loan types	0.03	0.05	0.04	0.04
Total card balance	6,432	9,166	7,279	5,295
Total credit balance	38,455	35,735	37,930	20,201
Age bank relationship (years)	8.1	6.2	6.8	4.8

Notes: Data at the borrower-cohort level for the sample of *borrower nearing distress* with two or more card lenders. Selective means that the borrower defaults on at least one card while repaying at least one other card. Selective largest balance means that borrowers defaulted on the cards that accounted for more than 50% of their total card balance measured in the month when they missed a payment. Selective smallest balance means that borrowers defaulted on the cards that accounted for less than 50% of their total card balance measured in the month when they missed a payment. Wage (formal) refers to wages recorded in administrative data (RAIS) and only available for workers employed in formal jobs. Wage (reported) refers to wages reported by banks. Formal sector is a dummy that takes the value 1 if the individual has a formal job. Debt service to income is the ratio of debt payments to income. All monetary amounts are in Brazilian reais (BRL).

Table 5: Descriptive statistics of regression variables

	Mean	SD	p25	p50	p75	N
Default	0.45	0.5	0	0	1	174,893
Positive global limit available	0.16	0.37	0	0	0	174,893
Limit utilization 50% - 60%	0.02	0.15	0	0	0	174,893
Limit utilization 60% - 70%	0.02	0.15	0	0	0	174,893
Limit utilization 70% - 80%	0.03	0.17	0	0	0	174,893
Limit utilization 80% - 90%	0.03	0.18	0	0	0	174,893
Limit utilization 90% - 100%	0.11	0.31	0	0	0	174,893
Limit utilization 100%	0.69	0.46	0	1	1	174,893
Digital bank	0.23	0.42	0	0	0	174,893
State-owned bank	0.11	0.31	0	0	0	174,893
Payroll loan	0.06	0.23	0	0	0	174,893
Mortgage	0.01	0.1	0	0	0	174,893
Auto loan	0.02	0.14	0	0	0	174,893
Personal loan	0.12	0.32	0	0	0	174,893
Other loan types	0.16	0.37	0	0	0	174,893
Default payroll loan	0	0.03	0	0	0	174,893
Default mortgage	0	0.01	0	0	0	174,893
Default auto loan	0	0.03	0	0	0	174,893
Default personal loan	0.01	0.07	0	0	0	174,893
Default other loan types	0.01	0.08	0	0	0	174,893
Positive card limit	0.31	0.46	0	0	1	174,893
Largest positive card limit available	0.16	0.36	0	0	0	174,893
Largest card balance	0.31	0.46	0	0	1	174,893
2nd largest positive card limit available	0.08	0.28	0	0	0	174,893
2nd largest card balance	0.31	0.46	0	0	1	174,893

Notes: Data at the borrower-bank-cohort level for the sample of *borrower nearing distress* with two or more card lenders and that default on a subset of their cards. All variables are binary.

Table 6: Predictors of selective default

	(1)	(2)	(3)	(4)	(5)	(6)
	Selective		Selective: less than 50%		Selective: more than 50%	
	All cohorts	Post May 2021	All cohorts	Post May 2021	All cohorts	Post May 2021
Limit utilization 50% - 60%	0.06***	0.06***	0.02	0.02	0.10***	0.11***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Limit utilization 60% - 70%	0.06***	0.06***	0.02	0.02	0.12***	0.14***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Limit utilization 70% - 80%	0.07***	0.08***	0.03**	0.03**	0.12***	0.14***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Limit utilization 80% - 90%	0.11***	0.12***	0.04***	0.04***	0.17***	0.19***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Limit utilization 90% - 100%	0.13***	0.13***	0.07***	0.06***	0.16***	0.18***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Limit utilization 100%	0.40***	0.41***	0.35***	0.34***	0.34***	0.37***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Positive global limit	-0.33***	-0.24***	-0.23***	-0.18***	-0.18***	-0.16***
	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.02)
Digital bank	-0.18***	-0.14***	-0.12***	-0.08***	-0.15***	-0.14***
	(0.00)	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)
State-owned bank	0.14***	0.13***	0.13***	0.11***	0.05***	0.05***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Payroll loan	-0.06***	-0.07***	-0.04***	-0.05***	-0.04***	-0.05***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Mortgage	-0.12***	-0.14***	-0.09**	-0.10*	0.02	0.01
	(0.04)	(0.04)	(0.05)	(0.05)	(0.04)	(0.05)
Mortgage x state-owned bank	0.29***	0.24***	0.25***	0.22***	0.07	0.03
	(0.04)	(0.05)	(0.05)	(0.06)	(0.04)	(0.05)
Auto loan	-0.05***	-0.05***	-0.04***	-0.05***	-0.06***	-0.06***
	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)
Personal loan	0.04***	0.03***	0.00	-0.01	0.04***	0.04***
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Other loans	-0.08***	-0.07***	-0.04***	-0.04***	-0.09***	-0.09***
	(0.00)	(0.01)	(0.01)	(0.01)	(0.00)	(0.01)
Default payroll	0.17***	0.17***	0.23***	0.23***	0.04	0.09
	(0.05)	(0.06)	(0.06)	(0.08)	(0.05)	(0.06)
Default mortgage	0.12	0.21	0.21	0.39	0.10	0.66***
	(0.20)	(0.32)	(0.22)	(0.26)	(0.23)	(0.07)
Default auto loan	0.12*	0.07	0.13	0.05	0.12**	0.11
	(0.06)	(0.08)	(0.08)	(0.10)	(0.05)	(0.07)
Default personal loan	0.21***	0.21***	0.20***	0.21***	0.05**	0.06**
	(0.02)	(0.03)	(0.03)	(0.04)	(0.02)	(0.03)
Default other loans	0.12***	0.06**	0.13***	0.10**	0.03	0.01
	(0.02)	(0.03)	(0.03)	(0.04)	(0.02)	(0.02)
Largest limit available	-0.05***	-0.04***	0.03***	0.03***	0.02**	0.02***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
2nd largest limit available	-0.02***	-0.02**	-0.00	-0.00	0.01	0.01
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Largest card balance	0.16***	0.15***	-0.33***	-0.32***	0.62***	0.57***
	(0.00)	(0.00)	(0.01)	(0.01)	(0.00)	(0.01)
2nd largest card balance	0.06***	0.06***	0.20***	0.19***	-0.01***	-0.01**
	(0.00)	(0.00)	(0.01)	(0.01)	(0.00)	(0.00)
Constant	0.18***	0.19***	0.20***	0.21***	0.14***	0.16***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Observations	174,893	120,962	78,021	52,594	96,872	68,368
R-squared	0.277	0.290	0.376	0.359	0.564	0.531

Notes: The sample is comprised of *borrowers nearing distress* who have two or more credit card lenders, default on at least one card, and repay at least one card. The data are at the borrower-cohort-bank level. All variables are binary. All regressions include borrower-cohort fixed effects. Robust standard errors are reported in parentheses. Selective less than 50% means that borrowers defaulted on the cards that accounted for less than 50% of their total card balance measured in the month when they missed a payment. Selective more than 50% means that borrowers defaulted on the cards that accounted for more than 50% of their total card balance measured in the month when they missed a payment.

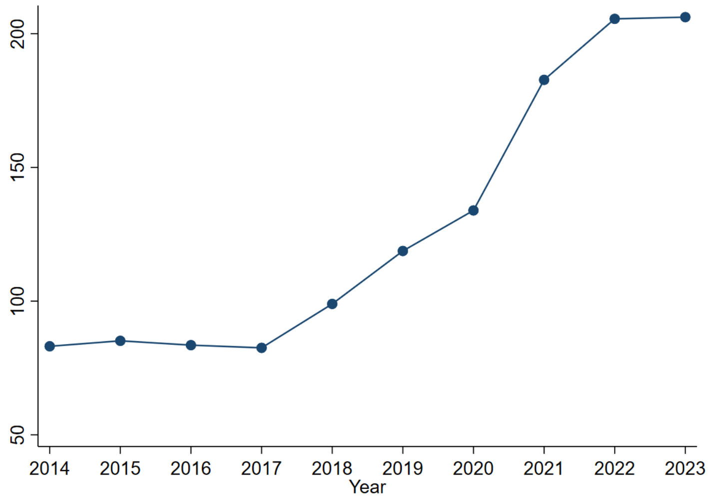
Online Appendix

Contents

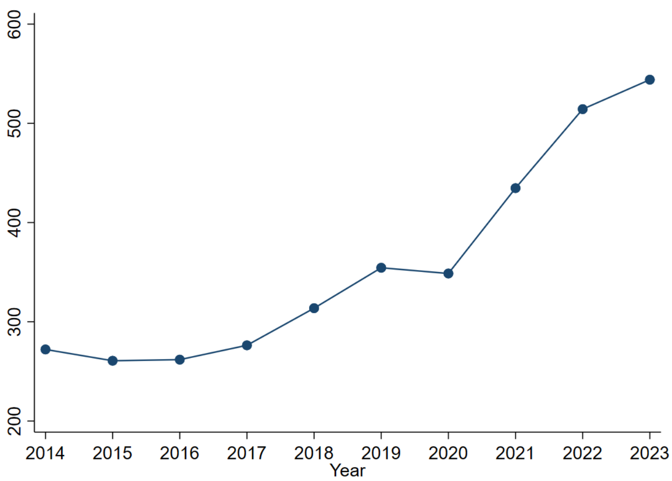
A Empirical part

A1 Institutional setting: additional details from aggregate data

Figure A1: Evolution of number of credit cards and credit card balance



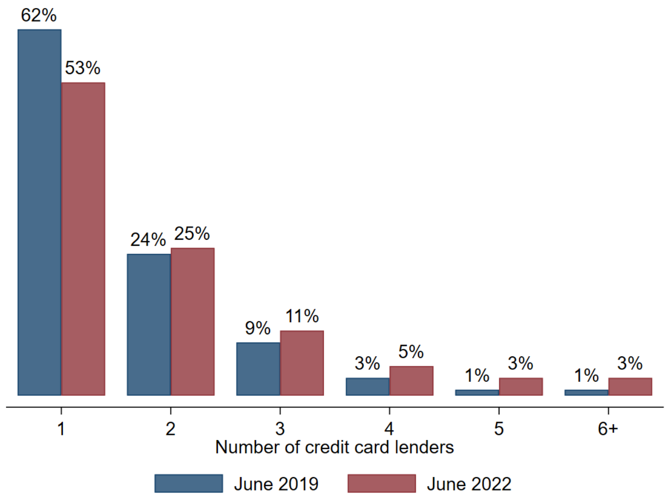
(**a**) Total number of active credit cards (in millions)



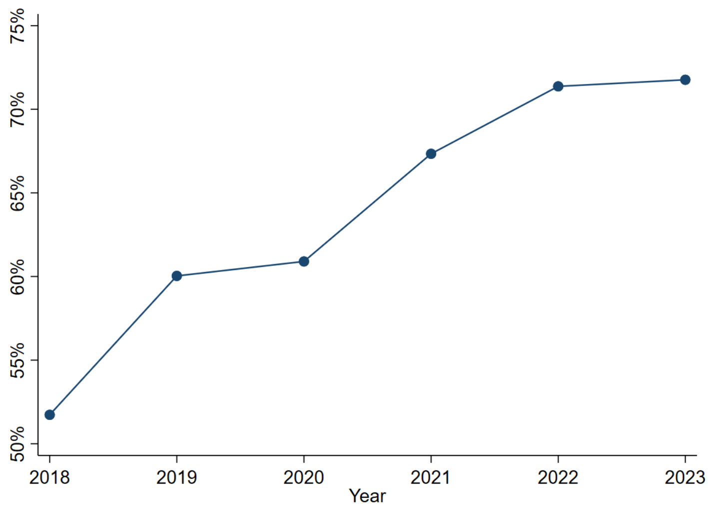
(**b**) Total credit card loans (inflation-adjusted values in billions BRL)

Notes: Total number of active cards and credit cards outstanding balance per year. Monetary quantities are in 2023 values, adjusted by the consumer price index (IPCA). Source: Central Bank of Brazil (*Estatísticas de Meios de Pagamentos* and IF.data).

Figure A2: Evolution of borrowers with more than one card lender



(**a**) Share of card users by number of credit card lenders, 2019 and 2022



(**b**) Share of total credit card balance held by borrowers with multiple card lenders

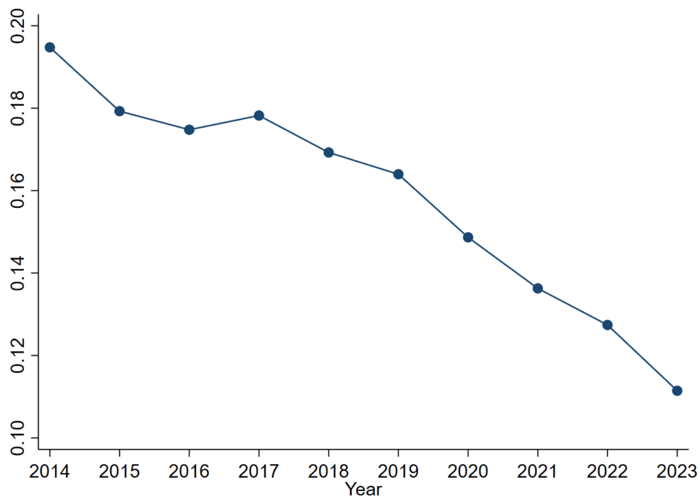
Notes: Computed based on figures from Central Bank of Brazil (2022).

Table A1: Number of credit card users per institution type

	June 2019	June 2022	Growth
Five largest banks (branch-based)	51.0	57.7	13%
Other banks and retailers	19.5	22.7	16%
Digital banks	8.9	36.5	310%
Credit unions	2.0	3.6	80%
Total	81.4	120.5	

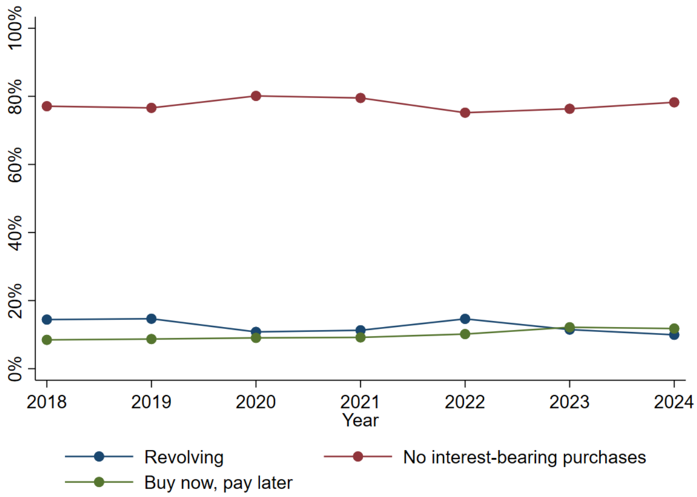
Note: Categories are not mutually exclusive. One individual can have accounts in more than one institution type. Source: Central Bank of Brazil (2022).

Figure A3: Herfindahl–Hirschman index: credit card loans

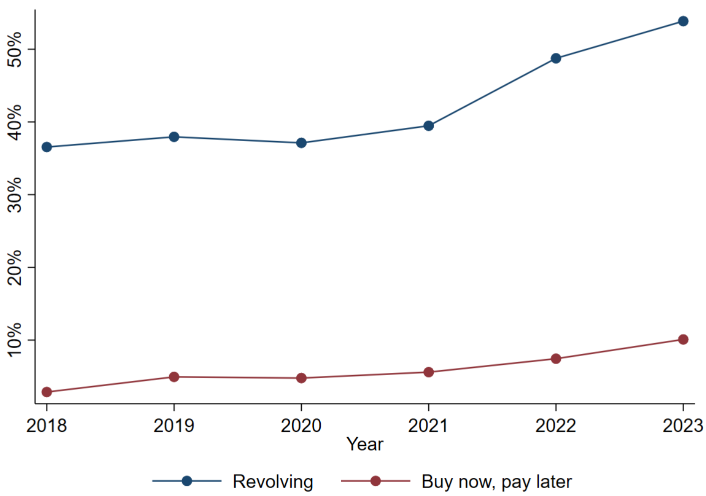


Notes: We calculate the Herfindahl–Hirschman Index (HHI) for the Brazilian banking sector based on the shares of each bank in the credit card loan market. We utilize data from IF.data (Central Bank Od Brazil).

Figure A4: Composition of credit card balance



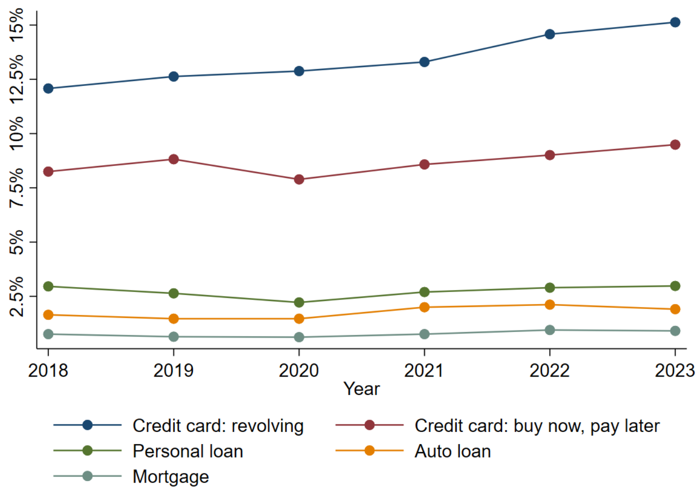
(**a**) Composition of credit card balance



(**b**) Share of total balance in default

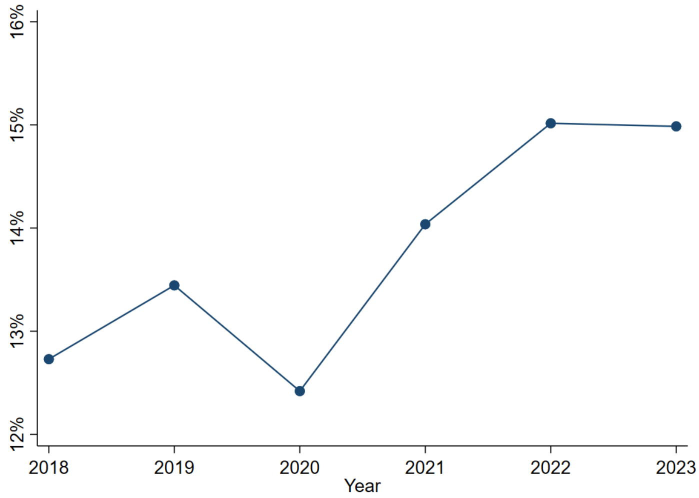
Notes: Balance in default refers to payments that are more than 90 days overdue. We use data from the Time Series Management System of the Central Bank of Brazil.

Figure A5: Monthly interest rate of different loan types

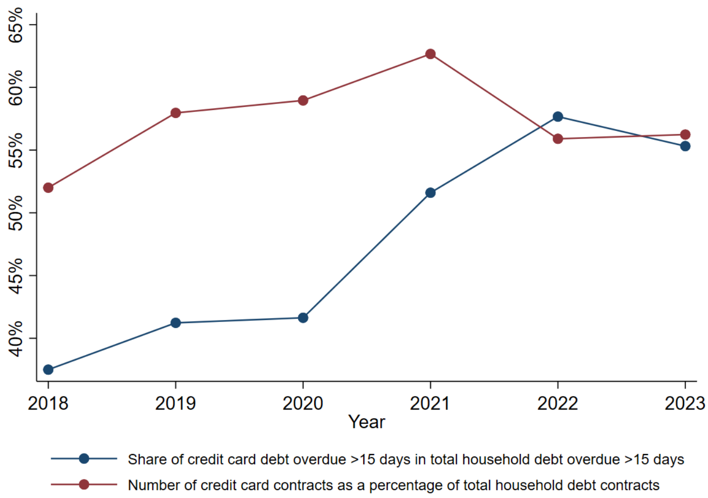


Notes: We use data from the Time Series Management System of the Central Bank of Brazil.

Figure A6: Participation of credit card debt in total household debt



(**a**) Participation in total household debt



(**b**) Participation in total amount overdue for more than 15 days and number of contracts

Notes: We use data from SCR.data of the Central Bank of Brazil.

A2 Sample procedure and characteristics

Table A2: Sampling procedure for cohort t

	Repayment Standing at t-1	Repayment Standing at t	Share sampled
Borrowers nearing distress	Good	Bad	1%
All other borrowers	Good	Good	0.025%
All other borrowers	Bad	Good	
All other borrowers	Bad	Bas	

Notes: Good repayment standing means that all cards are with a maximum delay of 15 days. Bad repayment standing means that at least one card is overdue for more than 15 days. The sample is taken at time t at the individual level, conditional on the individual having at least one active card.

Table A3: Default and delinquency incidence at $t-1$ (month before the cohort)

	Borrowers nearing distress		All other borrowers	
	Card level	Borrower level	Card level	Borrower level
Panel A: All card users				
Delinquency rate	0	0	10.3	13.2
Default rate	0	0	7.0	9.2
Panel B: Card users with 2+ cards				
Delinquency rate	0	0	10.4	17.6
Default rate	0	0	7.0	12.2

Notes: A credit card loan is delinquent (in default) if it is more than 15 (90) days overdue. A borrower is delinquent (in default) if at least one card is more than 15 (90) days overdue.

Table A4: Sample evolution

Cohort	Borrowers nearing distress	All other borrowers	Total
201906	15,186	15,799	30,985
201909	15,435	16,186	31,621
201912	13,348	16,476	29,824
202003	18,293	16,469	34,762
202006	10,128	16,390	26,518
202009	10,323	16,514	26,837
202012	11,281	16,931	28,212
202103	17,696	17,047	34,743
202106	15,804	18,025	33,829
202109	17,780	18,726	36,506
202112	22,750	19,668	42,418
202203	22,817	20,130	42,947
202206	20,407	20,667	41,074
202209	21,499	21,125	42,624
202212	20,097	21,536	41,633
Total	252,844	271,689	524,533

Notes: Number of cohort-individual observations per cohort and treatment status.

Table A5: Average number of relationships

	Number of bank credit relationships				Dummy
	All loans	Cards	All loans, except cards	Digital banks	2+ cards
Panel A: Population					
201906	2.61	1.54	1.07	0.14	0.36
201909	2.63	1.56	1.07	0.17	0.37
201912	2.67	1.60	1.07	0.18	0.38
202003	2.65	1.57	1.08	0.18	0.37
202006	2.64	1.54	1.10	0.20	0.35
202009	2.64	1.56	1.08	0.22	0.36
202012	2.69	1.60	1.09	0.26	0.37
202103	2.74	1.62	1.12	0.30	0.39
202106	2.79	1.66	1.13	0.40	0.4
202109	2.84	1.70	1.14	0.47	0.41
202112	2.90	1.76	1.14	0.52	0.43
202203	2.99	1.79	1.20	0.62	0.44
202206	3.04	1.82	1.22	0.67	0.44
202209	3.10	1.84	1.26	0.73	0.45
202212	3.14	1.86	1.28	0.79	0.46
Panel B: Treated subsample					
201906	2.85	1.75	1.10	0.20	0.46
201909	2.88	1.80	1.08	0.22	0.47
201912	2.92	1.84	1.08	0.19	0.49
202003	3.01	1.94	1.07	0.19	0.53
202006	2.83	1.83	1.00	0.17	0.48
202009	3.04	1.88	1.16	0.21	0.5
202012	2.96	1.87	1.09	0.26	0.48
202103	3.08	1.97	1.11	0.34	0.52
202106	3.11	1.97	1.14	0.43	0.52
202109	3.26	2.07	1.19	0.52	0.54
202112	3.19	2.07	1.12	0.54	0.53
202203	3.60	2.32	1.28	0.76	0.6
202206	3.62	2.32	1.30	0.83	0.59
202209	3.75	2.32	1.43	0.92	0.59
202212	3.72	2.28	1.44	0.95	0.58

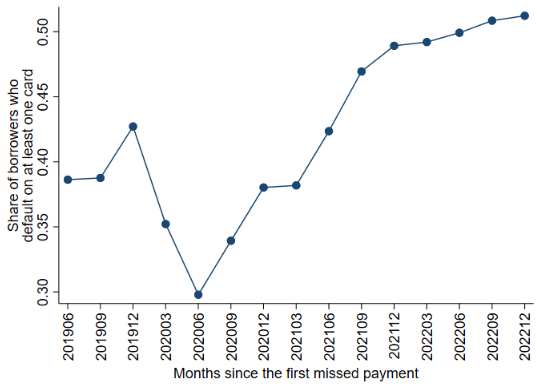
Notes: Data at the borrower-cohort level. We compute population averages using inverse probability weighting. Bank credit relationships refer to relationships with an active loan. Card relationships refer to the number of active credit card lenders. Digital banks refer to banks that operate without physical branches or agents.

Table A6: Size of groups based on the number of credit card lenders

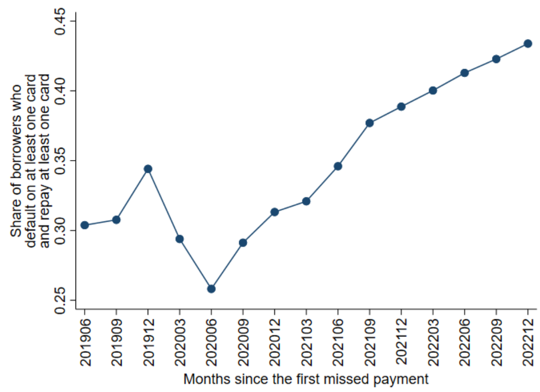
	Frequency			Shares	
Number of card lenders	Borrowers nearing distress	All other borrowers	Total	Borrowers nearing distress	All other borrowers
1	117,788	163,114	280,902	47%	60%
2	68,462	65,961	134,423	27%	24%
3	34,509	25,363	59,872	14%	9%
4	16,432	9,908	26,340	6%	4%
5	7,793	4,033	11,826	3%	1%
6+	7,860	3,310	11,170	3%	1%
Total	252,844	271,689	524,533	100%	100%

Notes: Borrowers are categorized based on the number of credit card lenders and the total number of lenders (across all products). Data at the borrower-cohort level.

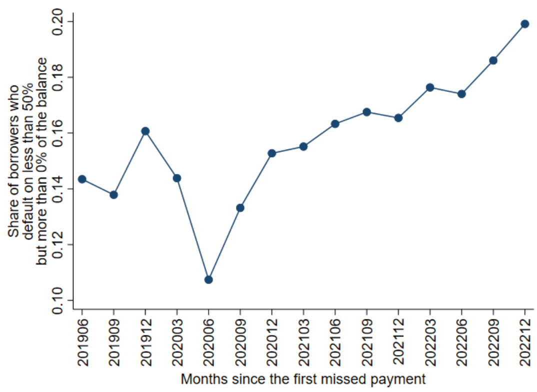
Figure A7: Default and selective default evolution, borrowers with at least two cards



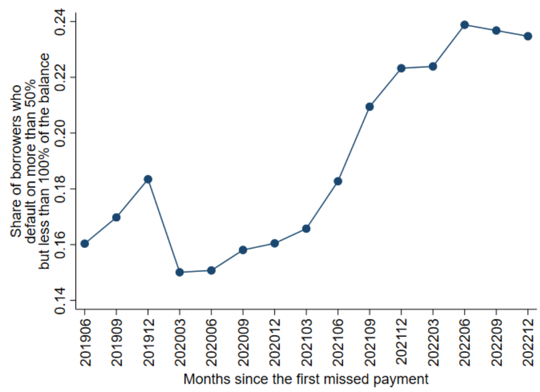
(**a**) Share of *borrowers nearing distress* with at least one card 90+ days overdue



(**b**) Share of *borrowers nearing distress* with at least one card 90+ days overdue and at least one in good standing



(**c**) Share of *borrowers nearing distress* with at least one card 90+ days overdue, at least one in good standing, and default on less than 50% of the total card balance



(**d**) Percentage of *borrowers nearing distress* with at least one card more than 90 days overdue and at least one card in good standing, and default on more than 50% of the total card balance

Notes: The sample is comprised of individuals in the treated group who have two or more cards. We classify a card as in default if there is balance overdue for more than 90 days six months after the borrower becomes more than 30 days overdue.

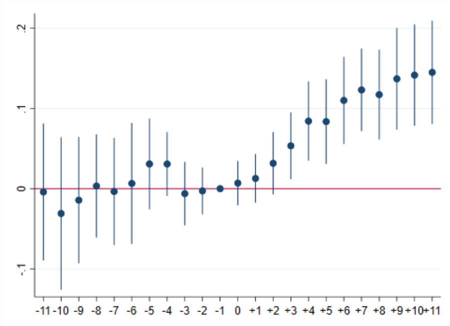
B DiD analysis

Table A7: Regression

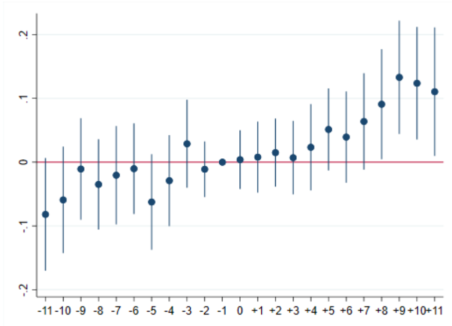
	(1)	(2)	(3)	(4)	(5)	(6)
	Number of cards	Logarithm card debt	Debt service to income	Card debt to income	Delinquency	Default
$Post\_Treat_{i,t}$	0.09***	0.09**	0.016	0.164***	0.015**	0.007
	(0.03)	(0.04)	(0.011)	(0.049)	(0.007)	(0.006)
Constant	1.32***	7.86***	0.035***	1.431***	0.027***	0.026***
	(0.01)	(0.03)	(0.005)	(0.040)	(0.006)	(0.004)
Individual FE	Yes	Yes	Yes	Yes	Yes	Yes
Firm×Month FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	65,590	65,590	65,590	65,590	65,590	65,590
R-squared	0.77	0.83	0.48	0.84	0.62	0.63

Notes: The figures present the results from estimating Equation **??**. Delinquency (default) are dummy variables equal to one if the individual has any debt payment overdue by more than 15 (90) days. In all regressions, the post-treatment window has a length of eleven months. Standard errors are clustered at the individual level.

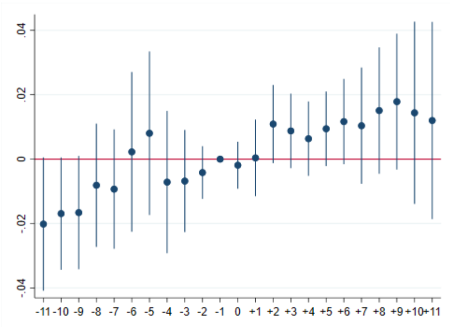
Figure A8: Difference-in-Differences



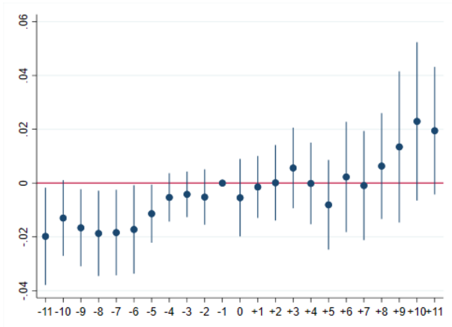
(**a**) Number of cards



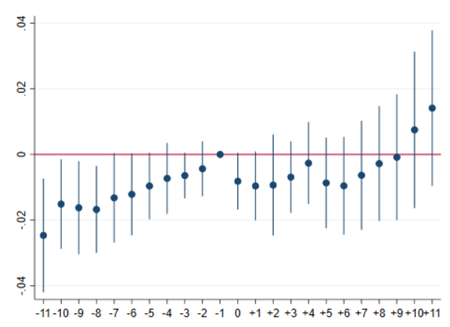
(**b**) Ln(card debt)



(**c**) Ratio debt service to wage



(**d**) Delinquency rate



(**e**) Default rate

Notes: The figures present the results from estimating Equation 1. We report 95% confidence intervals calculated using standard errors clustered at the individual level. Delinquency (default) are dummy variables equal to one if the individual has any debt payment overdue by more than 15 (90) days.

C Examples of micro-foundations for V_c

Let us now discuss different functional forms for V_c that can capture the stylized facts presented in section 3.2. The empirical section suggests that three main channels can affect repayment behavior. First, the lenders can invest in an app or a reward system that the borrower values, making the threat of cancelling the card upon default more costly. Second, the lender can change the credit limit on the card depending on the repayment behavior. Third, the lender can offer another loan product in the second period if the borrower does not default on the card.

Those three channels can be captured using the following parametrization of V_c:

$$\text{App or reward system: } V_c(q_c, \mathcal{R}) = q_c(\mathcal{R}) \tag{33}$$

$$\text{Credit limit: } V_c(\tilde{L}_c, \tilde{R}_c, \mathcal{R}) = \tilde{L}_c(\mathcal{R}) - \beta \tilde{R}_c(\mathcal{R}) \tag{34}$$

$$\text{Other Loan product: } \sum_c V_c(\underbrace{\tilde{L}_c, \tilde{R}_c}_{q_c}, \mathcal{R}) = E_{\varepsilon_b}[\max_b \{\tilde{L}_b(\mathcal{R}) - \beta \tilde{R}_b(\mathcal{R}) + \varepsilon_b\}] \tag{35}$$

In equation 33, q_c can be interpreted as the app quality or the generosity of the reward system. In equations 34 and 35, $\tilde{L}_c$ and $\tilde{R}_c$ are the loan size and the debt face value for the credit card and the other loan product, respectively. The variables q_c, $\tilde{L}_c$ and $\tilde{R}_c$ are functions of $\mathcal{R}$. That is, lenders can condition their value based on the borrower's repayment behavior, potentially canceling access to the bank app or the rewards if the borrower does not repay the credit card.

The timing assumption behind equations 34 and 35 is the following, and requires adding one period to the model. After seeing the repayment in period 2, the lender changes the credit card contract to offer a new loan product. The repayment $\tilde{R}$ of the new contract happens at the end of a third period. The benefit of borrowing $\tilde{L}$ happens in the second period. For simplicity, we assume that the borrower cannot get a new credit card contract in period 2 (for equation 34), or that the other loan contracts are exclusive in the second period (for equation 35). This assumption is in line with the empirical evidence that other contracts offered are mortgages or car loans, for which borrowers typically only get one. The fact that the borrower does not get a new card (for equation 34) can be micro-founded by a model in which the repayment shocks in the third period are perfectly correlated with those in the second period. In that case, since the next section (6.1) shows that the borrower repays only one contract in the second period, the lender learns that if they have seniority in the third period, and are thus the only lender.

The following section 6.1 shows that when lending generate a positive net present value, there exist an equilibrium in which it is optimal to exclude borrowers in the second period (i.e., set $q_c = L_c = R_c = 0$ if $c \notin \mathcal{R}$) if they did not repay their debt, and to set $\tilde{R} = \tilde{W}$ if they repays, where $\tilde{W}$ is the borrower wage in the third period. In that case, all three equations are equivalent: They are all increasing functions of one variable (q_c or $\tilde{L}_c$) conditional on repayment. V_c can be normalized to zero conditional on defaulting on card c. This justifies the general approach used in equation 6.

D Examples of Micro-foundations for c_c

Using the three micro-foundations discussed in section 5.1, potential functional forms for the cost function are:

$$\text{Rewards: } c_c(q_c, \theta_c(R_c, q_c)) := c_c(max\{q_c(\mathcal{R})\}) \text{ positive convex and increasing} \tag{36}$$

$$\text{Credit limit: } c_c(R, \tilde{R}, \tilde{L}, \theta_b(R, \tilde{R}, \tilde{L})) = -\theta_b(R, \tilde{R}, \tilde{L})\{\tilde{R} - mc\tilde{L}\} \tag{37}$$

$$\text{Other Loan: } c_c(R, \tilde{R}, \tilde{L}, \theta_c(R, \tilde{R}, \tilde{L})) = -\theta_c(R, \tilde{R}, \tilde{L})\tilde{D}(\tilde{R}, \tilde{L})\{\tilde{R} - mc\tilde{L}\} \tag{38}$$

In equation 36, we consider that setting a generous reward system requires the lender to set aside some money in the first period $c(max\{q_c(\mathcal{R})\})$. The operator $max\{q_c(\mathcal{R})\}$ captures the fact that the bank pays the cost in the first period and should ensure to have invested enough to provide the maximum reward it committed to.

In equations 37 and 38, the costs are paid in the second period, after the repayment $\mathcal{R}$ is observed. We drop the $\mathcal{R}$ from the variables $(R, \tilde{R}, \tilde{L})$ to simplify the notation. The cost function can be positive or negative depending on the markup the lender sets for the other product, sold in the second period. In equation 38, $D(\tilde{R}, \tilde{L})$ is the probability of choosing bank c in the second period. It is derived from equation 35.

As in section 5.1, when lending is a positive NPV, it is optimal to set $\tilde{R} = \tilde{W}$, where $\tilde{W}$ is the borrower's income in the second period. All the equations are thus functions of one variable (q or $\tilde{L}$).

E Proofs

A1 Proof exclusive contracts equilibrium

With assumption A1, default is not optimal. In addition, A1 also implies that it is optimal to max out the borrower's borrowing capacity (i.e., $R = W$). To see this, one can use the change of variable $R = \alpha L - u + V_c(q)$, and allow the lender to choose (u, L, q). The first-order condition with respect to L is always positive, so we must be at a corner solution.

More formally, let us denote u the borrower utility and $q := (\bar{q}, \underline{q})$ where $\bar{q}$ is the card quality conditional on repaying the card $\underline{q} \geq 0$ is the reward conditional on default:

$$u := \alpha L + \int_{-\infty}^{\bar{\epsilon}} V(\bar{q}) - R - \epsilon dF(\epsilon) + \int_{\bar{\epsilon}}^{\infty} V(\underline{q}) dF(\epsilon) \tag{39}$$

With $\bar{\epsilon} := V(\bar{q}) - V(\underline{q}) - R$. $\bar{\epsilon}$ is the threshold above which the borrower chooses to default.

The lender problem is:

$$\max_{\bar{q}, q, R, L} D(u)[\int_{-\infty}^{\bar{\epsilon}} R dF(\epsilon) - mcL - c(\bar{q}, \underline{q})] \tag{40}$$

Using the change of variable $L = \frac{1}{\alpha}[u - (\int_{-\infty}^{\bar{\epsilon}} V(\bar{q}) - R - \epsilon dF(\epsilon) + \int_{\bar{\epsilon}}^{\infty} V(q) dF(\epsilon))]$, the lender problem becomes:

$$\max_{\bar{q}\geq 0, q\geq 0, W\geq R\geq 0, u} D(u)\{\int_{-\infty}^{\bar{\epsilon}} RdF(\epsilon) - \frac{mc}{\alpha}[u - \int_{-\infty}^{\bar{\epsilon}} V(\bar{q}) - R - \epsilon dF(\epsilon) - \int_{\bar{\epsilon}}^{\infty} V(q)dF(\epsilon)] - c(\bar{q}, \underline{q})\} \tag{41}$$

This setup is general enough to capture the credit limit and new loan cases by letting: $c(q) = \int_{-\infty}^{\bar{\epsilon}} \tilde{c}(\bar{q})dF(\epsilon) + \int_{\bar{\epsilon}}^{\infty} \tilde{c}(\underline{q})dF(\epsilon)$.

The derivative with respect to $\underline{q}$ yields:

$$A := -V'(\underline{q})Rf(\bar{\epsilon}) + \frac{mc}{\alpha}V'(\underline{q})[1 - F(\bar{\epsilon})] - c_2(\bar{q}, \underline{q}) \tag{42}$$

$$A < 0\ \forall q, R > 0, \bar{\epsilon} \implies \underline{q} = 0 \tag{43}$$

The derivative with respect to $\bar{q}$ yields:

$$V'(\bar{q})Rf(\bar{\epsilon}) + \frac{mc}{\alpha}V'(q)[F(\bar{\epsilon})] - c_1(\bar{q}, \underline{q}) \tag{44}$$

The derivative with respect to R yields:

$$B := F[1 - \frac{mc}{\alpha}] - Rf(\bar{\epsilon}) \tag{45}$$

$$B > 0\ \forall R, \bar{\epsilon} \implies R = W \tag{46}$$

The derivative with respect to u yields:

$$\pi := [\int_{-\infty}^{\bar{\epsilon}} RdF(\epsilon) - mcL - c(q, \bar{q})] = \frac{D}{D'}\frac{mc}{\alpha} \tag{47}$$

The condition $\alpha - mc(\theta) > 0,\ \forall\theta$ implies that $F[1 - \frac{mc}{\alpha}] > 0$ in equation 45. Using the first order conditions with respect to q, we have $\bar{q} > \underline{q}$. Together with the conditions $V_c(q, \{c\}) - \epsilon_{ir} - R > V_c(q, \emptyset),\ \forall R \leq W_i, \forall\epsilon_{ir} \in S$, we have $f(\bar{\epsilon}) = 0$ if $R < W$. Equation

46 is satisfied so that $R = W$.

The same conditions imply that equation 43 is satisfied as well ($[1 - F(\bar{\epsilon})] = 0$ and $f(\bar{\epsilon}) = 0$, $c' > 0$ so that $B < 0$) so that the card is canceled conditional on not repaying.

Rearranging the first order conditions with respect to u (equation 47) and $\bar{q}$ (equation 44) using the fact that $F = 1$ and $f = 0$, $\frac{D}{D'} = \varepsilon^{-1}$ gives us the L^* and q^* conditions.

A2 Proof non-exclusive contracts equilibrium

We start by describing the behaviour of borrowers and lenders when borrowers own multiple cards. Then, we show that in equilibrium, borrowers indeed use multiple cards.

The proof proceeds in four steps. We first show that lenders cancel the card when the borrower does not repay the debt entirely. Then, we prove that the borrower chooses to repay only one card. Third, we demonstrate that the borrower uses multiple credit cards in equilibrium. Finally, we derive the equilibrium contract characteristics.

Step 1: *Lenders cancel the card conditional on not repaying the debt fully*

We show that the lender cancels the credit card if the borrower does not repay the full debt face value. As in proof A1, I denote $q = (\bar{q}, \underline{q})$, where $\underline{q}$ is the card quality conditional on default. We show that lenders optimally set $\underline{q} = 0$ if the borrowers repay less than the debt face value.

We prove the result here for the credit reward case (i.e., the cost is paid in the first period as in equation 36). Appendix A3 show that this result also holds for the situation in which the borrower also uses the credit card in period 2 (as in equations 37 and 38).

We now describe the timing in the lender problem 7 for the credit reward case. In period 2, the lenders can choose the card quality function $0 \leq q(R_c) \leq q^*$ at no cost. q^* is the maximum card quality, determined by the investment in the first period. R_c^* is the debt face value determined in period 1, R_c is the amount of debt c the borrower chooses to repay in the second period.

The lender maximisation problem 7, conditional on having originated a card, and having already invested in the card maximum quality (q^*) can be rewritten:

$$\max_{q(R_b)} \; E[R_b] \tag{48}$$

$$s.t.\ Borrower\ behaviour:\ \{R_b : \max_{R_c} \sum_c V_c(q_c(R_c)) - R_c,\ s.t. \sum_c R_c \leq W\} \tag{49}$$

$$Repayment\ lower\ than\ debt\ face\ value:\ R_b \leq R_b^* \tag{50}$$

$$Quality\ cannot\ be\ negative:\ q(R_b) \geq 0 \tag{51}$$

$$Maximum\ quality\ is\ bounded\ by\ period\ 1\ investment:\ q(R_b) \leq \bar{q} \tag{52}$$

To maximize the probability of repayment, the lender sets: $q_c(R_c) = \bar{q}$ for $R_c = R_b^*$ and $q_c(R_c) = 0$ otherwise. This is because they want to maximize $V_c'(q_c(R_c))q_c'(R_c)$. One way to do this is to set $q_c(R_c) = q^*$ when repaying fully and $q_c(R_c) = 0$ otherwise. This is the best response when other lenders also follow this strategy. This equilibrium may not be unique. However, this implementation prevents borrowers from repaying each credit card a bit, which is consistent with the data.

Step 2: *The borrower repays only one card when they borrow from multiple lenders*

Assumption A1 ensures that lenders set $R^* = W$ as long as $F[1 - \frac{mc}{\alpha}] - Rf(\bar{\epsilon}) > 0$ and that borrower do not default on all their cards (see appendix A1).

Given that $R_c^* = W$ for all cards c, and the step 1 results, the borrower chooses to default on all but one card.

Step 3: *Borrowers get multiple credit cards*

Assumption A2 ensures that the borrower gets more than one card. To see that, let us show that borrowers using only one contract cannot be an equilibrium.

We show in steps 2 and 3 that the borrower chooses to repay one card at most. Indexing by b the lender from which the borrower got their first card and c any other lender, the borrower can benefit from getting a new card from c with a loan size dL if:

$$\underbrace{\alpha L_b + W + V_b(\{c\},\{c,b\})}_{utility\ not\ repaying\ lender\ b} + \underbrace{\alpha dL - mc(\theta)dL + V_c(\{c\},\{c,b\})}_{utility\ new\ card} > \underbrace{\alpha L_b + W - R_b + V_b(\{b\},\{b\})}_{utility\ repaying\ card\ b\ and\ not\ getting\ new\ card} \tag{53}$$

Let us now consider that lender b offers the first-best contract so that the right-hand side of equation 53 is maximized,[18] which makes getting a new card less likely.

Lender b is unwilling to increase the loan size as this would trigger default, which increases the cost of lending and lowers the borrower's utility (see proof A1).

Yet, the borrower has incentives to get a new card from another lender. This happens because the new contract benefits both the borrower and the new lender, thanks to assumption A2 which makes equation 53 hold.

Indeed, the new contract offers the borrower the possibility to default on the first credit card while still having access to a second credit card at the end of the second period. A2 ensure that this possibility is valuable to the borrower so that the second lender can generate some profits by originating a new card: Formally, the marginal surplus generated by a dollar lent $(\alpha L_b + \alpha + W - mc(\theta))$ plus the cost of defaulting on one card only $(V_c(\{c\},\{c,b\}) + V_b(\{c\},\{c,b\}))$ is strictly greater than the utility of having only the first best contract $(\alpha L_b + W - R_b + V_b(\{b\},\{b\}))$.

The second lender offers a new contract if they expect the borrower to repay them with some strictly positive probability. In that case, there exists a repayment $(R = \frac{mcL+\epsilon}{\theta},\ \epsilon > 0)$ and loan size such that profits is positive $(D(u)[\theta R - mcL] > 0)$ and borrower utility increases when taking an additional card. Step 4 shows that such θ exists in a symmetric equilibrium.

Step 4: *Deriving the optimal loan size and quality*

[18]The same argument holds for any other contract offered. If $R < W$, then a competitor can make profits.

The borrower utility net of the card specific shocks ϵ_{ic} is denoted:

$$u := \sum_c L_c + \beta \sum_c \{\theta_c(V(q))[W - R_c + V_c(q,c) - E[\varepsilon_{ir}|\mathcal{R}=c] \tag{54}$$

$$+[1-\theta_c(V(q))]V_c(0,\mathcal{R}\neq c)\} \tag{55}$$

Where $V(q) := (V_c(q_c))_c$.

I normalize $V_c(c, \mathcal{R} \neq c)$ to zero. The maximization problem is thus:

$$\max_{u,q} D(u)[\theta_c(V(q))W - \frac{mc}{\alpha}[u - \beta \sum_d \{\theta_d((V(q))V_d(q_d) - E[\varepsilon_{ir}|\mathcal{R}=c]\}] - c(q)] \tag{56}$$

The first order conditions with respect to q yield:

$$V_c'(q_c)\theta_c'(V(q))W + \frac{mc\beta}{\alpha}\theta_c V_c'(q_c) = c'(q) \tag{57}$$

I used the fact that $\sum_d \theta_d'[V_d + E[\varepsilon|\mathcal{R}=c]] = 0$.

For simplicity, I also assumed that lenders do not internalize the effect of q on capital requirements. The rationale is that the government sets capital requirements, and those are taken as given by the lender.

A3 Repayment with other contracts in the second period

The section shows that it is optimal for the lender to cancel the card conditional on the borrower not repaying the full debt face value. We focus on the case in which the card quality is interpreted as offering a loan in the second period as in equations 37 and 38.

Formally, let us look at the lender problem in period 2 before the borrower makes his repayment choice. Lender b conditions the credit card offer in the second period on repayment r_b made by the borrower. Denoting the credit card offer $(L(r_b), R(r_b))$, the lender objective function is:

$$\max_{L(r_b),R(r_b)} \int r_b + [R_b(r_b)\theta((r_c)_c) - mcL(r_b)]dF(r_b) \tag{58}$$

$$s.t. F(r_b)\ defined\ by\ problem\ 60 \tag{59}$$

Let us assume that the borrower repayment shock ϵ are persistent for the first and second contract, so that lender b expects to lose seniority with probability one with the new loan contract if they lost it with the old one, that is, $\theta((r_c)_c)$ is zero if they repaid the other lender. This simplifies the exposition by making the second contract "exclusive" in the second period, as only one lender is willing to lend.

The borrower in the second period chose which card to repay. Their maximization problem is:

$$\max_{r:=(r_c)_c} W + \sum_c \alpha L_c(r_c) - R_c(r_c) - r_c + \epsilon_c \tag{60}$$

$$s.t. \sum r_c \leq W$$

$$0 \leq r_c \leq W$$

Suppose the other lenders offer $\bar{L}_c$, $R(r_b) = W$ if the borrower repays the debt in full and zero otherwise. We now show that doing this is the best response for lender b. The maximisation problem implies that the borrower repays only one card.

Using the borrower problem 60, the probability of lender b being repaid is thus: $Pr(\alpha\bar{L}_c + \epsilon_c \leq max_{r_b \leq W} W - \alpha L(r_b) - r_b + \epsilon_b)$. For the lender, the problem is then equivalent to choosing r_b and L instead of the pricing schedule. Lender utility is maximized at $r_b = W$ as $\alpha > mc$. One way to implement the optimal contract is by offering $\bar{L}_c$, $R(r_b) = W$ if the borrower repays the debt in full and zero otherwise. The lender problem is thus to choose the loan size conditional on repayment L_b. This problem is thus similar to the one analyzed in appendix A2. Formally, the lender problem is:

$$\max_{L_b} \overbrace{Pr(\alpha L_b + \epsilon_b \geq \alpha L_c + \epsilon_c \, \forall c)}^{\theta(L_b)}[W_1 + W_2 - mcL_b] \tag{61}$$

W_1 is the repayment from the first loan and W_2 from the second. The first order conditions give the optimal L_b.

F Cross-Selling

For the case in which a product is sold in the second period, we have:

$$c' := D'\pi + D[\sigma\tilde{W} - \tilde{m}\tilde{L} - \tilde{m}] \tag{62}$$

Plugging it into equation 14 , we have:

$$\frac{\beta}{\alpha}mc = D[\sigma(W - \tilde{m}L) - \tilde{m}] \tag{63}$$

$$\iff \tilde{L}^* = \frac{\tilde{W}}{\tilde{m}} - [1 + \frac{\beta}{\alpha}\frac{mc}{\tilde{m}}]\frac{1}{\sigma} \tag{64}$$

$\frac{1}{\sigma}$ is the markup absent cross subsidy. $\frac{\beta}{\alpha}\frac{mc}{\tilde{m}}\frac{1}{\sigma}$ is the cross-subsidy incentives: Having a credit card puts the lender in the borrower consideration set to get another loan.

Using equation 13, and the fact that $c(q^*)) = [1 + \frac{\beta}{\alpha}\frac{mc}{\tilde{m}}]\frac{D^*}{\sigma}$

$$L^* = \frac{\overbrace{W - c(q^*)}^{Lender\ break\ even\ condition}}{mc} - \overbrace{\varepsilon^{-1}}^{inverse\ demand\ semi-elasticity} \tag{65}$$

The borrower utility is thus:

$$u^* = L^* + \beta\tilde{L}^* = \alpha[\frac{W + [1 + \frac{\beta}{\alpha}\frac{mc}{\tilde{m}}]\frac{D^*}{\sigma}}{mc} - \varepsilon^{-1}] + \beta[]\frac{\tilde{W}}{\tilde{m}} - [1 + \frac{\beta}{\alpha}\frac{mc}{\tilde{m}}]\frac{1}{\sigma}] \tag{66}$$

$$\Longleftarrow u^* = \alpha[\frac{W + [1 + \frac{\beta}{\alpha}\frac{mc}{\tilde{m}}]\frac{D^*}{\sigma}}{mc} - \varepsilon^{-1}] + \beta[\frac{\tilde{W}}{\tilde{m}} - [1 + \frac{\beta}{\alpha}\frac{mc}{\tilde{m}}]\frac{1}{\sigma}] \tag{67}$$

The impact of cross-subsidy is:

$$\frac{\alpha}{mc}\frac{1}{N}\frac{1}{\sigma} + \frac{\beta}{\tilde{m}}\frac{1}{\sigma}[\frac{1}{N} - \frac{mc}{\alpha}] \tag{68}$$

Cross-subsidy can be a way to attract the borrower to make them on your choice set, this is bad for welfare. On the other hand this can allow the borrower to get more credit card loan as this provides incentives to lower the markup to attract borrowers.

G Exclusive versus non-exclusive

Let us interpret q^{NE} as an additional loan contract taken in period 2 and use the extreme value random shocks as in Section 6.2.3.

A1 Exclusive case

The borrower's utility in the exclusive case is:

$$u_i^E = \alpha L + \beta E[W - R + \{\max_c V(q) + \tilde{\sigma}\epsilon_c\}] \tag{69}$$

$$= \alpha L + \beta[W - R + ln(\sum_c exp(\tilde{\sigma}V(q_c))) \tag{70}$$

Assuming that lending a card in the first period puts the borrower in the consideration set for the second product, the profits are:

$$\max_{R,L,q} D(u)[R - \frac{mc}{\alpha}L + \tilde{D}(q)[W - mcq]] \tag{71}$$

$$\max_{u,q} D(u)[W - \frac{mc}{\alpha}(u - \beta ln(\sum_c exp(\tilde{\sigma}V(q_c)))) + \tilde{D}(q)[W - mcq]] \tag{72}$$

The first order condition with respect to q yields:

$$\frac{mc}{\alpha}\beta\tilde{\sigma}V'(q_c)\tilde{D}(q) + \tilde{D}'[W - mcq]] - \tilde{D}(q)mc = 0 \tag{73}$$

$$\Longleftrightarrow q^E = \frac{W}{mc} - \tilde{\varepsilon}^{-1}[1 - \frac{mc}{\alpha}\beta\tilde{\sigma}V'(q_c)] \tag{74}$$

The relevant functions at equilibrium are (when denoting N the number of lenders in the consideration set in the second period):

$$c(q) = -\frac{1}{N}[W - mcq] \tag{75}$$

$$V(q) := \gamma q - W - \tilde{R} = \gamma q \tag{76}$$

$$L^E = \frac{W - c(q^E)}{mc} - \epsilon^{-1} \tag{77}$$

$$q^E = \frac{W}{mc} - \tilde{\varepsilon}^{-1}[1 - \frac{mc}{\alpha}\beta\tilde{\sigma}\gamma] \tag{78}$$

$$u_i^E = \alpha L + \beta[\sigma V(q_c)) + ln(N)] \text{ in a symmetric equilibrium} \tag{79}$$

A2 Non-exclusive case

$$u_i^{NE} := \sum_{c \in B} \alpha L_c^{NE} + \beta E[\max_c\{\overbrace{E[\max_d V(q|c) + \tilde{\sigma}\epsilon_d]}^{V_c}] + \varepsilon_c\}] \tag{80}$$

$$= \sum_{c \in B} \alpha L_c^{NE} + \beta[ln(\sum_d exp(V(c)))] \tag{81}$$

The lender problem is:

$$\max_{R,L,q} D(u)[\theta R - \frac{mc}{\alpha}L + \theta_c \tilde{D}(q)[W - mcq]] \tag{82}$$

$$\max_{u,q} D(u)[\theta W - \frac{mc}{\alpha}(u - \beta[ln(\sum_d exp(V(c)))]) + \tilde{\theta}_c D(q)[W - mcq]] \tag{83}$$

Taking the first order condition with respect to q:

$$\theta' W + \frac{mc}{\alpha}\beta\tilde{\sigma}V'(q_c)\theta_c\tilde{D}(q) + [\theta(q)\tilde{D}'(q) + \theta'(q)\tilde{D}(q)][W - mcq]] - \theta(q)\tilde{D}(q)mc = 0 \tag{84}$$

$$\iff q^{NE} = \frac{W}{mc} \overbrace{-\tilde{\varepsilon}_{NE}^{-1}}^{lower\ markup} + \overbrace{\varepsilon_{NE}^{-1}\frac{mc}{\alpha}\beta\tilde{\sigma}V'(q_c)]}^{under-investment} + \overbrace{\frac{\theta'}{\delta}W}^{over-invesment} \tag{85}$$

with $\delta := \theta(q)\tilde{D}'(q) + \theta'(q)\tilde{D}(q)$.

The markup tend to be lower because of the competition for loan seniority.

The relevant functions are now:

$$c(q) = -\theta\frac{1}{N}[W - mcq] \tag{86}$$

$$V(q) := \gamma q \tag{87}$$

$$L^{NE} = \frac{\theta W - c(q^E)}{mc} - \epsilon^{-1} \tag{88}$$

$$q^{NE} = \frac{W}{mc} - \tilde{\varepsilon}_{NE}^{-1}[1 - \frac{mc}{\alpha}\beta\tilde{\sigma}\gamma] \tag{89}$$

$$u_i^E = \sum_c \alpha L_c + \beta[\sigma V(q_c)) + ln(N) + ln(|B|)] \text{ in a symmetric equilibrium} \tag{90}$$

A3 Comparison exclusive versus non exclusive

We have:

$$u_i^{NE} = \alpha L_c^{NE} \cdot |B| + \beta[\tilde{\sigma}V_c(q^{NE}) + ln(|B|)] \tag{91}$$

$$u_i^E := \alpha L_c^E + \beta\tilde{\sigma}V_c(q^E) \tag{92}$$

The welfare impact of non-exclusive contract are ambiguous. Non-exclusive contract benefits borrower due to their love for variety (captured by $ln(|B|)$). It also decrease the markup for the loan market in the second period due to the competition for loan seniority ($\varepsilon_{NE}^{-1} < \varepsilon^{-1}$). However, a non-exclusive contract creates a debt dilution problem, which can create distortions (i.e., the over-investment channel $\frac{\theta'}{\delta}W$).

H Identification of supply parameters and counterfactual

$$\max_{X,\xi} D(f(X)+\xi)\overbrace{[\tau + p_r \cdot [\theta(X)r - mc]L - mc^m(\xi)]}^{\pi(X,u)} \tag{93}$$

Using the change of variable $U = f(X) + \xi$, we have:

$$\max_{X,U} D(U)\overbrace{[\tau + p_r \cdot [\theta(X)r - mc]L + mc^m(U - \beta X)]}^{\pi(X)} \tag{94}$$

The first order condition with respect to X yields (assuming $mc(d, U - \beta X)$ linear):

$$rL: \; p_r \cdot [\theta + \theta' rL] = mc^m \beta^r \tag{95}$$

$$L: \tau - p_r mc + L\theta' r + mc^m \beta^L = 0 \tag{96}$$

Those two equations allow us to recover the marginal costs conditional on having default elasticities and revolving probabilities (mc^m for equation 95 and $\tau - p_r mc$ for equation 96).

Let us now look at how to solve for counterfactuals.

Using equation 96, we can express $L\theta' r$ as a function of parameters. Lets denote this γ. Plugging it into the equation 95 and using the fact that $\theta' = \sigma\theta(1-\theta)$, we can solve for R for each bank b. Using the formula for $\frac{\theta_b}{\theta_c}$, we can then recover L_c as a function of

L_b. Using the θ_b we can then solve for L_b.

Let us consider that we can solve for R and L. In that case the maximization problem is of the form:

$$\max_U D(U)[A - mcU] \tag{97}$$

Where A is a constant (derived below). The first order condition yield:

$$D'(U)[A - mcU] = Dmc \tag{98}$$

When D has a logit form, we have:

$$[A - mcU] = D[A - mcU] + mc \tag{99}$$

Indeed, the function $F(A - mcU) := D(U)[A - mcU] + mc$ is a contraction mapping as $D(U) < 1$.